

Academic Language and Mathematics: A Study of the Effects of a Content and Language-Integration Intervention on the Preparation of Secondary Mathematics Pre-Service Teachers

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Curriculum and Instruction

Academic Language & Mathematics:
A Study of the Effects of a Content and Language-Integration Intervention
on the Preparation of Secondary Mathematics Pre-Service Teachers

Dissertation

By

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ABSTRACT

Researchers have found that as students progress through school, the importance of language grows due to the content specificity that emerges, especially in the secondary grades, and due to the preparation of these students to enter adulthood once their schooling is completed. Even as students' instruction in various content areas becomes more in-depth and specialized, so does the terminology employed in the content. It is because of this specificity and union of language and learning that English-language learners' (ELLs') ability to comprehend and produce content-area academic language is crucial to their success. When questioning the quality of instruction ELLs are receiving in mathematics, the attention logically shifts to the pedagogical abilities of their teachers. However, historically, mathematics teachers have lacked language-acquisition knowledge and strategies necessary to adequately address the needs of linguistically diverse learners. In order to authentically promote and pursue quality mathematics education for all students, teachers of mathematics must be trained in recognizing the language demands of mathematics and in applying or developing strategies to address the nuances of the language in this subject area. The research in this study contributes to this work.

This dissertation documents the effects of an intervention, woven into a secondary mathematics methods course and designed to prepare mathematics teachers to support ELLs' content and language learning. The study was based on the assumption that mathematics is much more than computations, and thus, requires a shift in the how the role of the mathematics teacher is viewed. Both qualitative and quantitative empirical evidence regarding the intervention's influence on the participants' attitudes and preparedness to teach the academic language of mathematics were generated. Twenty-nine

students over the course of two years took part in this research. Five students from the second year were selected for an in-depth case study based on their range of experiences with learning other languages, interactions with linguistically diverse youth, and practicum placements for the subsequent spring semester. The larger group of preservice teachers was surveyed at the beginning and end of their enrollment in the course, and their course assignments were collected. In addition, case-study participants were interviewed at the start and completion of the semester, and their practicum-office submissions were examined. A framework to encourage pupils' acquisition of mathematical academic language is proposed. Essential outcomes indicate that the intervention not only affected the participants' beliefs and attitudes towards their own preparedness for teaching ELLs in mainstream mathematics classes, but also it imparted concrete strategies for the modification of teaching and learning experiences in the preservice teachers' future practices. The results of this study correlate to existing literature regarding linguistically responsive pedagogy and extend this theory by integrating language-acquisition strategies throughout a content-methods course for the middle- and high-school levels.

DEDICATION

I dedicate this dissertation in loving memory of both my paternal and maternal grandparents: Rosa and Louis Terrell, Sr., and Anthony and Arlene Polk.

I miss you all so much, but I hope I've made you proud.

I also dedicate this dissertation to my mother, Sandra Terrell, who always pushed me to reach for the stars and do whatever God placed in my heart. I may have done the work, but Mom, you've always been "the wind beneath my wings." I love you.

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There are so many to thank. First, I must give all glory and honor to my God, my Lord and Savior, Jesus Christ, without whom I really have no idea where I would be. In the immortal words of André Crouch, “Just let me live my life, and let it be pleasing, Lord, to Thee. And should I gain any praise, let it go to Calvary!” Dad, my life is Yours.

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Chapter 1

INTRODUCTION

Statement of the Problem

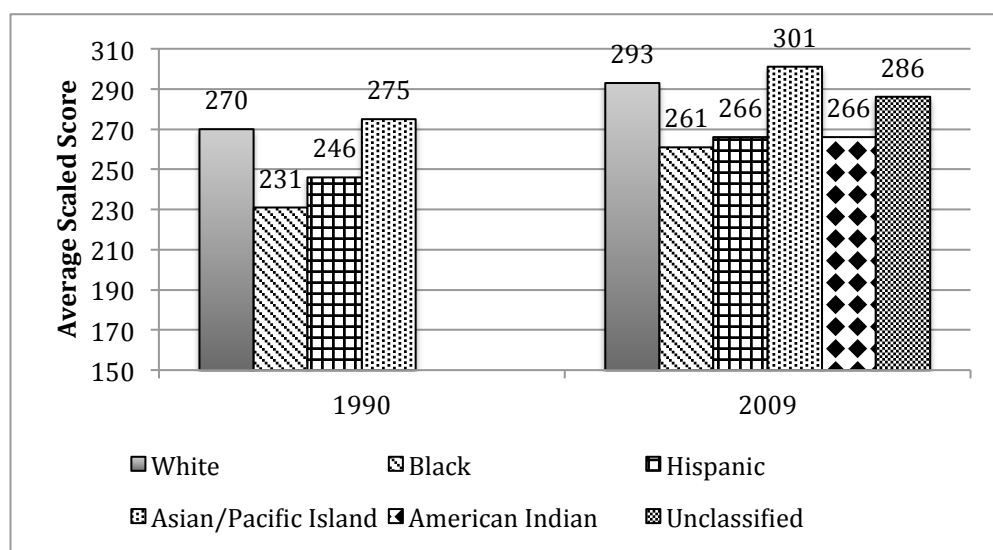
According to a study by Fortuny, Capps, Simms, and Chaudry (2009) on behalf of the Urban Institute, the demographics of school-aged children in the United States is changing and doing so rapidly. In 2007, more than one-fifth of these children (approximately 16.4 million children) had at least one immigrant parent, an increase from 13% to 23%. This count doubled from the eight million children of the same demographic in 1990. In 2006, children of immigrants comprised more than 10% of the total child population in 29 states, which is an increase from only 16 states in 1990. While the study by Fortuny and her colleagues indicates that half of children of immigrants live in California, Texas, and New York, it also notes that this segment of the American population is growing in almost every state.

The implication of such a shift in the population is that the change has began to reflect itself the public school, a microcosm of U.S. society. Thus, the twenty-first century public-school teacher's reality is that he or she will increasingly instruct more students whose home language is not American English (Brisk et al., 2002), and that "Teachers will need to have a clear understanding of the many ways that the hierarchy of cultures and languages in the larger society is reinforced in schools" (Beykont, 2002, p. xxiv). The demographics of those educated in our schools would not be of concern were it not for the decades-old existence of an academic-achievement gap in student performance between Caucasian students and disenfranchised students, comprised mostly of African Americans,

Latino Americans, and children with special needs primarily, but also including other students whose first language is not English.

The nationwide mathematics scores of the eighth graders who took the National Assessment of Educational Progress (NAEP) provide a glimpse at this gap (see Figure 1.1). According to these results in 1990, Black and Hispanic students scored approximately 14.4% and 8.9% lower than White students, respectively. However, in 2009, there was a decrease in the difference in White and Black students' scores to 10.9% (still significantly lower), but a slight increase in the difference between White and Hispanic students to 9.2%. The latter year's data also included American Indian students, who had an average scaled score that was about 9.2% lower than White students. These data indicate the perpetuation of achievement disparities among races and ethnicities.

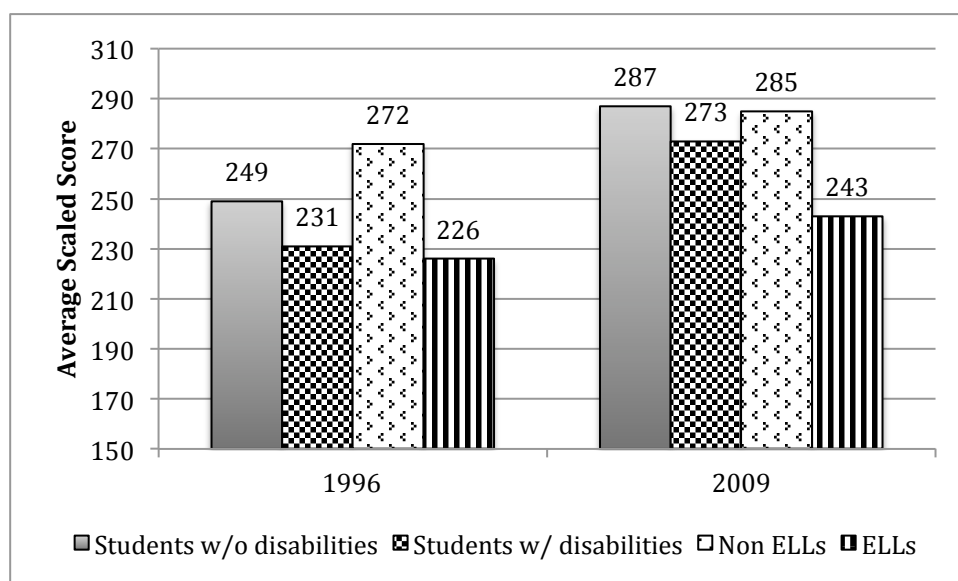
Figure 1.1. *NAEP Mathematics Score Comparison by Race/Ethnicity*



If the ability groups are examined, similar disparities in the eighth-grade mathematics scores for NAEP also arise (see Figure 1.2). The earliest data available was from 1996 for these groups, but the differences are still significant. For students with

disabilities, the average scaled scores are approximately 7.2% lower than those without disabilities in 1996 and then 4.9% in 2009, indicating a sizeable improvement but still a significant difference. However, the disparities between English-language learners (ELLs) and their counter parts are even more striking than those discussed for the various races and ethnicities. In 1996, the scores for ELLs were about 16.9% lower than non-ELLs. While there was slightly less difference in the 2009 scores, there was still a disparity of 14.7%, which is greater than that for Black students in 1990 compared to White students.

Figure 1.2. *NAEP Mathematics Score Comparison by Ability*

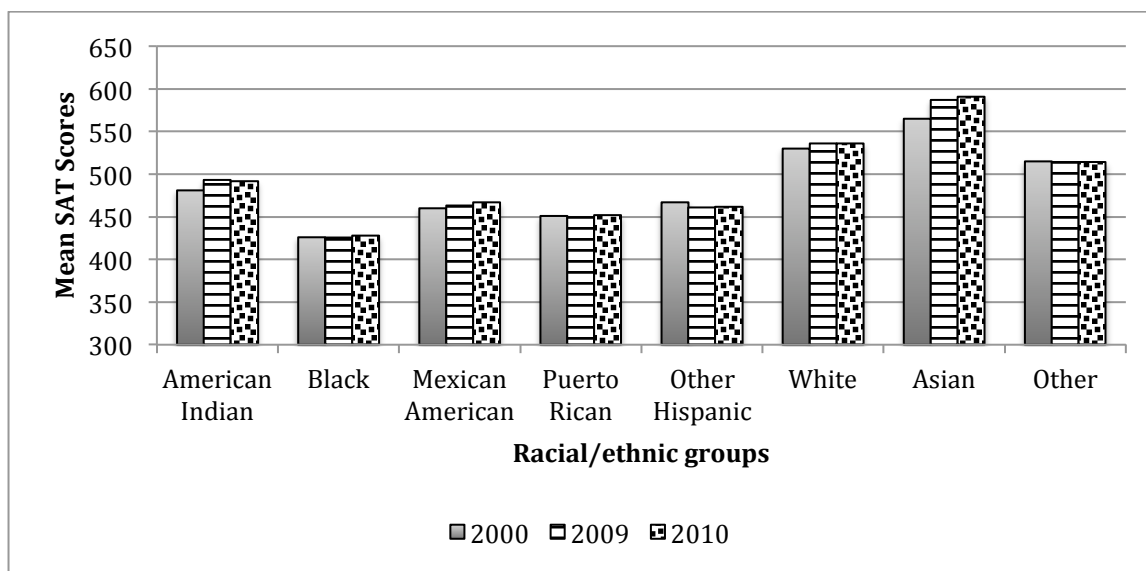


In examining the Scholastic Aptitude Test (SAT) scores for high-school seniors, the results for ELLs and other students of color does not look any more promising.

Figure 1.3 shows that Black students are clearly underperforming compared to the other ethnic groups, especially White and Asian students, by as much as 163 points (an approximately 38.1% difference). Mexican-American high schoolers are fairing a bit better with maximum deficits of 73 points (about 15.8%) compared to White students and 124 points (about 26.8%) to that of Asian students. The greater differences between Puerto

Rican students and White and Asian students range from 86 points (roughly 19.1%) to 139 points (roughly 30.8%), respectively.

Figure 1.3. *SAT Mean Scores by Ethnicity, Mathematics*



These data are only a sample of the massive amounts of data to which we can refer to support the claim that ELLs are not receiving equitable instruction in our schools.

Meanwhile, researchers have found that as students progress through school, the importance of language grows due to the content specificity that emerges, especially in the secondary grades, and due to the preparation of these students to enter adulthood once their schooling is completed. Even as students' instruction in various content areas becomes more in-depth and specialized, so does the terminology employed in the content. Schleppegrell (2007) purports, "Learning the language of a new discipline is a part of learning the new discipline; in fact, the language and learning cannot be separated" (p. 140). Bunch (2010) contends that it is because of this specificity and union of language and learning that ELLs' ability to comprehend and produce content-area academic language is crucial to their success. "Therefore, there is an urgent need for all teachers to develop

culturally sensitive and language appropriate instruction so that all students can succeed” (Dong, 2004, p. 202). It is these concerns that this dissertation attempts to address.

Purpose of Study

In order to consider the integration of language and mathematics, four assumptions were made by the researcher. The first is that mathematics is more than numeracy and computations. The second is that each content area has its own language consisting of much more than content-specific terminology. The third assumption is that language is a gatekeeper for access to higher-level coursework in mathematics, and finally, because of these three points, it must be concluded that mathematics teachers are also language teachers. It is these suppositions that drove the inquiry into how to begin attending to the needs of linguistically diverse mathematics students at the secondary level.

When questioning the quality of instruction ELLs are receiving in mathematics, the attention logically shifts to the skills of the mathematics teachers. However, historically, these teachers, in particular, have lacked language-acquisition knowledge and strategies necessary to adequately address the needs of ELLs (Bunch, 2010; Calderón, 2007; Dong, 2004; Tevebaugh, 1998), as well as any linguistic deficiencies of other diverse learners. Conversely, instructors of English as a second language rarely receive training and development in specific content areas (Calderón, 2007). If we acknowledge the National Council of Mathematics Teachers’ (NCTM, 2000) belief in “mathematics for all” and include the over five million school-aged ELLs (Office of English Language Acquisition, 2008) under this creed, then teachers of mathematics must be trained in recognizing the language demands of mathematics and in applying or developing strategies to address the nuances of the language in this subject area (Cardelle-Elawar, 1990; Secada, 1996).

Gebhard and her peers (2002) report, “too many teacher education program are still guided by the erroneous assumption that their job is to train teachers to work in ‘regular’ (monolingual English) settings, when in reality all classrooms have, or will soon have, students whose first language is not English” (p. 220)– a fact elucidated by the statistic that only one in four of all U.S. programs offer bilingual or ESL tracks. In addition, “An enormous amount of information is presented to preservice teachers education students. Not only are they learning their subject matter disciplines, they are also being introduced to a number of additional disciplines that related to learning, teaching, and schooling” (Richardson, 2002, p. 95). However, if university-based, teacher education programs endeavor to remain relevant to such an evolving society, Lucas, Villegas, and Freedson-Gonzalez (2008) offer, “Preservice teacher education programs can [*and must*] engage prospective teachers in various types of activities that will prepare them to learn about ELLs in their future classes” (p. 367). The research in this study will explore the effectiveness of an instructional intervention administered during a required secondary mathematics methods course in preparing undergraduate and graduate-level preservice secondary mathematics teachers to address the linguistic needs of ELLs. The intervention in this study was designed by a teacher educator in connection with a larger content-and-language integration study at the university. The intervention was initially implemented in the fall 2009 semester and later revised by the educator for the fall 2010 administration, who also included the investigator in the second round of implementation.

The study utilized “mixed” data sources (Greene & Caracelli, 2003), employing surveys, observations, interviews, and documents such as course assignments and other artifacts. The participants completed pre- and post-surveys and allowed the inclusion of

their course assignments and pre-practicum submissions as artifacts. “Purposeful” sampling (Creswell, 2003; Maxwell, 1992) was utilized in choosing a smaller contingent of five course participants for pre- and post-course interviews, with the criteria for selection including the participants’ experiences in working with ELLs, their own language backgrounds, and whether or not they would be involved in pre- or full-practicum experiences in the spring semester of the same academic year. In addition, pertinent classes were observed and recorded.

Research Question(s)

Halliday (1978) refers to the term *register* as “a set of meanings that is appropriate to a particular function of language, together with the words and structures which express these meanings” (p. 195). Thus, it can be concluded that the register for mathematics is the set of meanings that is appropriate for mathematics, including the words and structures that express these meanings, ideas, and concepts. Further, Moschkovich (2004) explicates, “the notion of register depends upon the situational use of much more than lexical items and includes phonology, morphology, syntax, and semantics as well as non-linguistic behavior” (p. 194). In a recent research review, Schleppegrell (2007) notes that one of the preeminent challenges in helping students acquire this specific register is to assist them in “mov[ing] from everyday, informal ways of construing knowledge into the technical and academic ways that are necessary for disciplinary learning” (p. 140). Thus, this study will seek to answer the following question and sub-questions:

How does a content-and-language integration intervention effect the preparation of secondary mathematics preservice teachers in order to improve instruction for diverse learners, more specifically ELLs, in mainstream

mathematics classes?

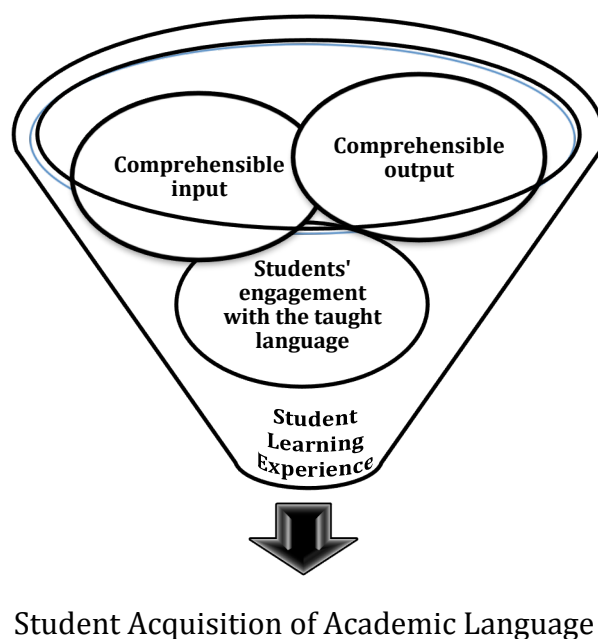
- a. Do the preservice teachers' beliefs change regarding their ability to provide content and language-integrated learning opportunities for ELLs in mathematics? If so, how?
- b. Do the preservice teachers' ability to recognize academic-language challenges for ELLs evolve during the secondary mathematics methods course? If so, how?

Conceptual Framework

Often, ELLs have trouble with processing and decoding language in their mathematics classes due to the lack of understanding their mathematics teachers possess in regards to language acquisition. Fillmore and Snow (2002) claim, "The U.S. teaching force is not well equipped to help these children and those who speak vernacular dialects of English adjust to school, learn effectively and joyfully and achieve academic success" (p. 7). Research also has shown that the teachers of ELLs, in particular, tend to possess little classroom experience and often lack basic credentials, let alone more specialized expertise in mathematics (Abedi & Herman, 2010). Therefore, a resulting assumption is that the instruction of ELLs, even for those pupils in transitional language courses, is regularly lacking the combined language and mathematics content necessary for academic success. Thus, the preparation of content teachers must include the subject matter as well as language-acquisition theory and strategies – preferably integrated with the subject matter, for modeling purposes.

Gay (2000) defines culturally responsive pedagogy as the use of “cultural knowledge, prior experiences, frames of reference, and performance styles of ethnically diverse students to make learning encounters more relevant to and effective for them” (p. 29). Through this asset-based lens, students’ differences are affirmed and used as tools through which their educative experiences can be enhanced, and teachers assume the role of cultural brokers, assisting their pupils in navigating the nuances of the new culture to which they are being introduced. It is this kind of mediation that is also necessary to aid ELLs in their acquisition of English – not just conversational, but primarily academic. Lucas and colleagues (2008) state, “Although teachers whose primary responsibility is to teach students subject matter cannot be expected to become experts on language, they can learn to identify and articulate the special characteristics of the language of their disciplines and make these explicit to their ELLs” (p. 365), an instructional methodology deemed *linguistically responsive pedagogy* (LRP). Within LRP pupils are taught the key language used in a particular content area and given opportunity to engage with the language through social interactions. To further extend LRP, students should then be required to produce or output the language in some observable way in order for teachers to make the appropriate alterations to future instruction, but also to build their own awareness of linguistic structures and meanings (Swain, 1985). Figure 1.4 represents my interpretation of a student’s learning experience as it encompasses this extension of LRP and combines, or “funnels” together, these three core ingredients – teacher’s comprehensible input (Krashen, 1981), student engagement (Lucas et al., 2008), and student’s comprehensible output (Swain, 1985).

Figure 1.4. *Producing Student Acquisition of Academic Language, Extending LRP*



The integration of LRP into content instruction begins with teacher educators and the constructs of courses designed to prepare future instructors (Lucas et al., 2008). Teacher educators themselves need to receive professional development in how to include this theme in their courses. From this enhancement, teacher educators can emphasize the quality of language input and output in the classroom setting, design their courses to investigate subject-specific academic language, and model language analysis for their course participants. It bears noting that this analysis is much more involved than skimming a text and highlighting the bold-faced vocabulary terms, but rather, in order to address the mathematics register, it includes sentence constructs, language structures, and idioms that would impede an ELL from comprehending the tasks assigned within the text. Also, directives such as *paraphrasing*, *justifying*, and *summarizing* must be deconstructed to determine the exact product that students are expected to submit and to determine the

explicit instruction that may be necessary to assist students in producing the desired output. Finally, Lucas and her peers (2008) suggest that teacher-education programs require a course dedicated to the theme of teaching ELLs, a course where preservice teachers can practice differentiating their instructional plans for ELLs, and observational or practicum experiences in schools well-populated by ELLs.

Lucas and Grinberg (2008) purport that teachers of ELLs should be prepared for encounters with these students in several language-related areas – personal experiences, attitudes and beliefs, knowledge, and skills. For the latter area, instructors need to acquire skills in conducting linguistic examinations of oral and written language, communicating across students’ cultural and linguistics differences, and designing instruction that addresses both language and content, as research is escalating in its emphasis on the learning of content and language simultaneously rather than in isolation (Bay-Williams & Herrera, 2007).

Importance of the Study

The research of this dissertation documents the elements of an methods course infused with a language intervention designed to prepare mathematics teachers to support ELLs, resulting in empirical evidence about its influence on preservice teachers’ attitudes and preparedness to teach the concepts and academic language of mathematics to ELLs. According to Lucas and colleagues (2008), “Far too many new teachers find themselves unprepared to meet the special challenges of teaching academic content to ELLs” (p. 371). Beykont (2002) adds,

“A major but often overlooked challenge of U.S. education programs concerns the preparation of predominantly White, middle-class, English monolingual preservice

teachers to teach a student body whose home cultures are not only different from the school culture but also low in status in the wider society” (p. xxiv).

Considering the recent Common Core State Standards (2010), it is required that “mathematics instruction for ELLs should address mathematical discourse and academic language” (p. 2), which substantiates the necessity of preparing current and future mathematics teachers in discourse and language theory and strategies. In addition, Shanahan and Shanahan (2008) deduce that in order to address the literacy instruction for middle and high school students, the curriculum for the preparation of the teachers of these students must be reformulated. This study provides insight into one particular preparatory model for secondary preservice mathematics instructors to engage all students in the language-related aspects of the subject area.

Lucas and her peers (2008) assert, “To be effective, today’s teachers need a broad range of knowledge and skills, including deep content knowledge, pedagogical content knowledge, knowledge of how children and adolescents learn in a variety of settings, skills for creating a classroom community that is supportive of learning for diverse students, knowledge about multiple forms of assessment, and the ability to reflect on practice” (p. 362). Beykont (2002) continues that these skills must also include the “develop[ment] of culturally responsive and respectful classroom that connect what students already know to what they need to learn” (p. xxiii). This investigation will, therefore, provide empirical evidence about how preservice mathematics teachers’ pedagogical knowledge of academic language in their content area assists them in creating learning environments that support both students’ language and content acquisition.

This study will use survey data to capture preservice secondary teachers' attitudes and perceptions of their preparedness to teach ELLs and the role of the mathematics instructor in the language-acquisition process; their opinions about ELL inclusion in mainstream classes; their knowledge about modifications and accommodations for ELLs; and their philosophies regarding the teaching, learning, usefulness, and nature of mathematics. In addition, the survey has a drawing portion that asks participants to describe their own experience as a learner in a mathematics classroom and then to illustrate their future classrooms. The study then continues with selecting a purposeful sample of these preservice teachers and interviewing them at the beginning of semester to gain more insight into each of their personal histories and the survey results, then interviewing them again at the end of the semester to assess in more detail these teachers' sense of preparedness and probe into some of their responses in course assignments as well as their post-survey results.

There has been little empirical research concentrating on the methods and strategies required at the preservice level for the successful instruction of ELLs (Lucas & Grinberg, 2008), and an even scarcer amount that gear these methods towards secondary mathematics. In addition, of the existing literature, few studies have featured the instruction of an entire class of content-specific prospective teachers regarding language acquisition and instruction. Rather, most existing studies have been very targeted case studies. Therefore, this multi-layered approach will provide a comprehensive analysis of the effects of the course-infused intervention on the preparation of preservice secondary mathematics teachers at this particular university, thereby offering valuable insights to other teacher-preparatory institutions considering similar models. This study is one

component of a larger, multi-pronged content-and-language integration study, supported by a federal Title III grant and conducted at a catholic university located in the northeastern portion of the United States.

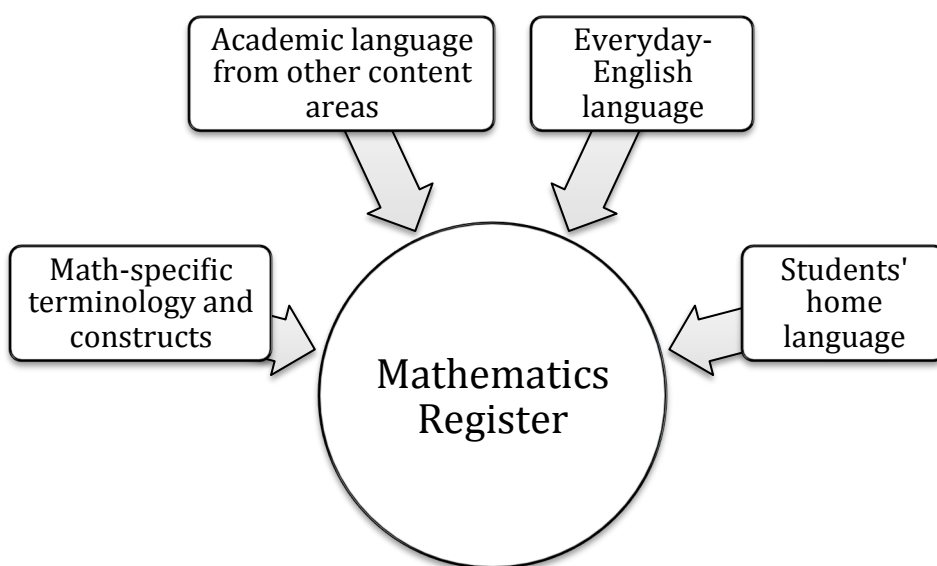
Definition of Terms

For the purpose of this study, three key terms will be defined for clarity and understanding of the research perspective: *academic language*, *mainstream*, and *mathematics register*. Charlot and O'Malley (1994) define *academic language* as “the language that is used by teachers and students for the purpose of acquiring new knowledge and skills” (p. 40). They assert that this language is used for “imparting new information, describing abstract ideas, and developing students’ conceptual understanding.” Also referred to as “the language of schooling” (Schleppegrell, 2004), a student’s fluency in this form of language can lead to increased academic achievement and can even influence a student’s relationships with his or her teachers and other academic faculty.

In an earlier section of this chapter, the mathematics register was defined as the set of meanings conveyed through language that is appropriate for mathematics. Some might define mathematics register as the academic language required to function successfully in a mathematics setting or classroom, and thus mathematics register, and academic language in or for mathematics will be used interchangeably throughout this dissertation. It should be noted that this type of language is comprised of mathematics-specific terminology and linguistic constructs, but it is also influenced by academic language from other content areas, everyday English vernacular, and students’ home languages – be they English dialects or other languages altogether. Figure 1.5 illustrates my interpretation of the composition of mathematics register. Note that none of the arenas are limited to “a list of

technical words and phrases” (Moschkovich, 2002, p. 195). Such an “interpretation reduced the concept of mathematical register to vocabulary and disregards the role of meaning in learning to communicate mathematically.”

Figure 1.5. *Composition of Mathematics Register*



Finally, the term *mainstream* is employed throughout this dissertation. It is used to refer to English-speaking classrooms in schools within the United States that provide all or most of their instruction in English. Classes specifically geared towards students with learning or other disabilities are not included in this category. Also, while advanced or “honors” level courses can be included under this umbrella, College Board Advanced Placement (AP) classes are also excluded from the title of *mainstream*.

Researcher Positionality

This dissertation study was birthed out of my own experience as a classroom teacher. I taught high-school mathematics for four years in an urban, low SES, mostly underrepresented minority school and then middle-school mathematics for two years in a

suburban school in which the student population was predominantly Caucasian and upper-middle class. While the schools were drastically different in the demographics of the families serviced as well as the design and cultures of the schools, one aspect that was consistent in my experiences was the need to address the language demands of my content area. Initially, I thought the language issue in the courses I taught was about the mathematics-specific vocabulary, but as my classes evolved to be populated by more limited-English proficiency (LEP) and former LEP students, it became evident that there was something larger at work. Thus, I promised myself that I would look further into language acquisition when I returned to graduate studies as a doctoral student.

Rossman and Rallis (2003) recommend, "Rather than pretending to be objective, state and make clear you are and what assumptions drive the study" (p. 36). Despite this statement, as I began the observations for this study, I tried to remain as objective an onlooker as possible for the purposes of validity. However, I very quickly fell into the position of a participant observer, "interact[ing] personally with participants during activities in the natural setting in order to build empathy and trust and to further [my] understanding of the phenomenon" (Gall et al., 2010, p. 349). This especially occurred with the interview participants with whom I interacted with on a continuous basis beyond the course. Within each observation, I found that my role and discussions shifted between documentarian, mathematics teacher educator, language coach, and teacher mentor. This resulted in an unexpected amount of reciprocity (Lather, 1986; Rossman & Rallis, 2003), wherein in exchange for their participation in the study, the preservice teachers were given access to my experience and advice. I believe this exchange resulted in enriched research

experiences for “both researcher and researched” (Lather, 1986, p. 263) and in the illumination of implications for future studies.

Overview of the Dissertation

This study consists of seven chapters. Chapter One presents the overarching framework of the research. It outlines and situates the problem and presents the research questions to be investigated, and also provides the rationale for this study. A literature review is provided in Chapter Two, and discusses multilingualism theory as well as social-justice theory, including more specifically stereotype-threat theory. It is through these lenses this dissertation research is justified. Chapter Three outlines the methodology and approaches utilized in this research, specifying and justifying the quantitative and qualitative data to be collected and the procedures for analysis.

Chapters Four, Five, and Six analyze the data and report findings for this study. Chapter Four details the findings from the pre- and post-survey, gathered from both the fall 2009 and fall 2010 semesters. Chapter Five presents portraits of the case-study participants chosen from the fall 2010 semester, while Chapter Six is an in-depth analysis of the five participants, discussing themes and assertions that emerge (Creswell, 1998). Finally, Chapter Seven returns to the research questions and utilizes the findings of the study to answer them. This final chapter will discuss implications for teacher education, specifically for secondary mathematics, limitations of the study, and recommendations for future research.

Chapter 2

REVIEW OF LITERATURE

Gee (1999) proposes that although “we always actively use spoken and written language to create or build the world of activities... thanks to the workings of history and culture, we often do this in more or less routine ways” (p. 11), resulting in the separation of activities and institutions from language. This perspective may be key in understanding how the role of language in schools has historically been based in politics and social status (Clarkson, 1992). In many countries the language in which a school operates is selected by a colonial power in order to promote unity, although the region is usually comprised of various cultures and languages. In Papua, New Guinea, “Schools were seen as a way of training an elite who would one day lead the country and also as a way of bringing unity into such a diverse land” (p. 417). Education in the United States has served similar purposes as well.

Torres-Velasquez and Lobo (2004/5) purport, “Learning to use any language is a complex activity, even when it is your native language” (p. 255). According to Gibbons (2003), part of language learning is also becoming adept at selecting the appropriate language for particular contexts, making language a semiotic system. To achieve “fluency” in mathematics, students must become adept at making these choices in their speaking, writing, and computations within this content area, as well as become proficient in navigating among these communicative modes. Despite this, mathematics has historically been considered to consist merely of computations; thus, causing aptitude in language skills to be deemed unnecessary for success in the subject. It must be added, however, that

numerical skill and the comprehension of sporadic mathematics vocabulary are not enough for pupil success in mathematics, especially when problem-solving examples are introduced. In these instances, students require more than just computational prowess, but also command of syntax, semantics, lexicon, register, and other grammatical units (Celedón-Pattichis, 2004; Domínguez, 2005; Secada, 1996; Tevebaugh, 1998; Fillmore & Snow, 2002). Additionally, students need to be able to translate mathematics terminology into symbolic form prior to executing any calculations to reach a solution and to make subsequent conclusions (Cardelle-Elawar, 1990; Domínguez, 2005). Unfortunately, the inclusion of these language skills in the mathematics classroom is lacking, primarily due to the void of language tutelage for mathematics teachers which leaves them ill-prepared to incorporate these skills into their instructional practices. As a result, “many teachers find these [language] problems to be a hassle, an interruption to ‘normal’ teaching, and an unwelcome burden” (Tevebaugh, 1998, p. 215). This study will, thus, address and analyze the preparation of a group of preservice secondary mathematics teachers, and determine the affects of their preparation on their instructional methods, specifically around academic language.

The review of literature that follows is comprised of three major sections. In the first section, the theoretical framework of this study is outlined and examined. This section includes an analysis of key theorists’ contributions as well as of some of the most current literature regarding language and mathematics. The second portion of the review will focus on empirical investigations regarding language and mathematics as well, specifically those involving English-language learners (ELLs). The final section of the review will be an interpretive summary of the previous sections. This chapter will assist in formulating the

research questions of this study within the larger body of related literature.

Theoretical Framework

This section will examine the literature of two theoretical arenas: teaching ELLs and social justice. The research on teaching ELLs will incorporate theories on how ELLs acquire language, dilemmas teacher experience while instructing ELLs, and literacy-based strategies for teaching mathematics that are connected to teaching ELLs. The social justice portion of the theoretical framework will consider the experiences of ELLs in American schools historically, currently, and typically in the content area of mathematics. It will also highlight methods the research suggests for more linguistically inclusive classrooms.

Teaching English-language learners

Barwell (2009) describes *multilingualism* as not only “the presence of two or more languages” (p. 2) in a classroom, but also the overt utilization of two or more languages within classroom affairs. Within a typical classroom in the United States, English would be considered one of these languages. In addition and especially at the secondary education level, there would exist the academic language of the content area. Then, finally, for a growing number of students in the United States, there exists at least one other language that is spoken at home. For some, this language would be considered a dialect or slang, but for others, it is the primary language of another country altogether. Thus, a secondary mathematics classroom can be considered a breeding ground for the study of multilingualism.

When a child is acquiring a second language, research shows that the ease with which the child will do so is based upon the proficiency he or she attains in the first language (Celedón-Pattichis, 2004). Further, “If students have not developed the language

used for academic tasks in their first language, they may experience difficulty with CALP [cognitive/academic language proficiency] in their second language” (p. 205-206). Clarkson (1992) even notes that students who attain competence in both languages have academic advantages as their knowledge of the two languages support one another. In mathematics, children who possess this expertise actually outperform children documented as native and monolingual (Clarkson & Galbrath, 1992; Secada, 1996). Therefore, it can be concluded that while students need to attain proficiency in academic language, there is no reason for them cast aside one language for another (Fillmore & Snow, 2002). Rather, students should be taught new language in a way that values the language that they bring into school and takes advantage of the linguistic aptitude that students already possess (Feldman, 2002).

When children are learning mathematics and working with word problems that are not written in their first language, two primary matters arise: children’s processing time required to make connections between the linguistic structures of the language they are reading and their first languages and to comprehend what a word problem is asking (Celedón-Pattichis, 2004). One of the reasons more time may be called for is that words children use in their natural environments often have different meaning in a mathematical context – again, the issue of mathematical register (Schleppegrell, 2007). Celedón-Pattichis cites the following example from her study: “A number 300 can holds $13 \frac{7}{8}$ ounces. A number 2 can holds 28 ounces. How many more ounces does a number 2 can hold than a number 300 can?” (p. 207). The author found that students interpreted the word *can* as a verb rather than a noun, and thus, they used 300 and 2, the labels of the cans, in their calculations rather than $13 \frac{7}{8}$ and 28, the number of ounces the cans could hold.

Therefore, while pupils may receive general reading instruction, attention must be paid to the comprehension skills required for success in mathematics (Calderón, 2007).

The challenges of any immigrant in a new country are numerous, to say the least. For a nonnative child to attend a school in a language that he or she may have little to no exposure to is a daunting undertaking, especially in a country such as the United States wherein the acceptance of non-English languages in the classroom can be sparse. According to Fillmore and Snow (2002), “Children discover very quickly that the only way they can have access to the social or academic world of schools is by learning the language spoken there” (p. 18). In addition, research has shown that high concentrations of these students often go to schools that have trouble with academic achievement and have higher dropout rates (Abedi & Herman, 2010). Language modifications and accommodations are typically left to the sole task of ELL specialists or bilingual teachers, rather than integrated throughout the mainstream educational experience, resulting in an inadequately schooling experience for linguistically diverse students (Calderón, 2007; Cummins, 1997; Fillmore & Snow, 2002). Thus, diversity evades the school culture, and “the interactions that pupils experience... are unlikely to promote either academic growth or affirmation of pupil identity” (p. 112). Alternatively, “When all language arts, ESL, special education, and content teachers in a school work together, more students achieve” (Calderón, 2007, p. 35).

The National Council of Teachers of Mathematics (NCTM, 2000) promotes the acquisition of mathematical knowledge for all students in the United States. The organization also notes that immigrant students may require assistance in order for this acquisition to take place.

“Students who are not native speakers of English, for instance, may need special attention to allow them to participate fully in classroom discussions. Some of these students may also need assessment accommodations. If their understanding is assessed only in English, their mathematical proficiency may not be accurately evaluated” (p. 13).

Research has shown, however, that “cultural and linguistic minority students have less exposure to content, and their instruction tends to cover less content relative to nonminority students” (Abedi & Herman, 2010, p. 727). Often, instruction for these pupils is altered by “diluting the course content, providing few modifications to the way they [the teachers] speak, and ignoring or excluding these students from class discussions and learning” (Dong, 2004, p. 202). The result, therefore, is the absence of linguistically diverse students in higher-level mathematics courses and teeming them disproportionately in lower-level courses.

While there is no exact course of action, or “magic cure,” for teaching this particular population of students (Beykont, 2002; Gebhard et al., 2002), research does cite the establishment and maintenance of high expectations for all students as a starting point for teachers in aiding students in their achievement and for the existence of equity in the content-area classroom (Brisk et al., 2002; Gebhard et al., 2002; NCTM, 2000). One might be inclined to think that first-language translations of course material, both verbal and written, would be a viable method to assist ELLs in learning the target language. However, research has demonstrated that the opposite is true (Fillmore, 1985). First-language translations have been found to stifle teachers’ inclination to utilize alternative access methods that would equip the students to learn the subject-matter content in English and

to participate in the dialogue of the class. In addition, ELLs have been found to ignore the English communication in the classroom and learning the language for themselves, knowing that eventually they will be provided the necessary information in their first language. Thus, language modifications and accommodations, rather than translations, are necessary for the development of both the ELL and the instructor.

Discourse & dialogue

Bay-Williams and Herrera (2007) report, “In mathematics education, research and national standards have highlighted the value of communication and discourse in the learning of mathematics” (p. 44). However, academic language is particularly demanding as it requires an exchange between contextualization and decontextualization of the language encountered in the subject are, as well as “a broad knowledge of words, phraseology, grammar, discourse structure, and pragmatic conventions for expression, understanding, and interpretation” (Fillmore & Snow, 2002, p. 28). It is because of such complexity that students must be given ample opportunity to engage in classroom activities and to regularly practice language through expressive communication – speaking and writing – i.e. discourse. However, note that Bay-Williams and Herrera (2007) differentiate between *communication* and *discourse*. If the latter is assumed to be a chain of words or sentences that convey a thought or idea (Moschkovich, 2002), then this implies that there is a level of relating that can or may occur either without or beyond words usage, i.e. “a ‘dance’ that exists in the abstract as a coordinated pattern of words, deeds, values, beliefs, symbols, tools, objects, times, and places” (Gee, 1999, p. 19) – *Discourse*. For the purposes of this review, the teaching and learning of “little d” (*discourse*) to ELLs and the nuances of dialogue, exchanging ideas utilizing discourse, will be focused upon.

Lucas and her peers (2008) suggest several principles, or “essential understandings” (p. 362), of which all teachers of ELLs should be aware. One of these principles is that there is a difference between conversational proficiency and academic-language proficiency and that the latter requires much more time and explicit instruction. The teacher is instrumental in bridging this gap by way of providing comprehensible input, as Krashen (1982) terms it, for pupils. However, Swain (2005) continues that students not only need to decipher the language that is being taught, but they also need to demonstrate their depth of their acquisition by producing the language in some way and becoming cognizant of their own communicative capabilities as a part of the learning process.

The product to which Swain (2005) refers to may be a verbal response to the teacher, collaborative dialogue among student peers, or a written piece, but the author argues that how one is taught (the *input*) is only a portion of the educative story. He also notes that the modes, or “functions” (p. 471), in which the output occurs may vary. Some students become aware of their linguistic prowess or deficiencies while attempting to express a thought in the newly acquired language: the “noticing/triggering function” (p. 474). Similarly, a student may attempt various ways of expressing an idea to enable the person or people with whom he or she is communicating to understand the idea. However, this function is considered “hypothesis testing” (p. 476) by Swain. The author’s third function, “metalinguistic (reflective)” (p. 478), refers to a student’s use of language “to reflect on language produced by others or the self;” thereby, necessitating a collaborative setting in which the student’s source of “solo mental functioning” has its roots in joint activities and communication. It could be interpreted that one progresses through these

linguistic functions, one's language prowess evolves from comprehension, to producing language for oneself, to responding to prompts, and finally, to full dialogue.

In response to Swain's notion of comprehensible output, Krashen (1998) argues that there is not enough evidence to support the output hypothesis. He notes that output of any kind in the classroom is too rare, that there exist *cases* of high levels of language acquisition devoid of any output scenarios, and that students develop high anxiety when forced to speak in languages they are acquiring. However, Swain does not promote output solely, but rather, both input *and* output, stating that input only is simply not enough.

The power of negotiating meaning as a part of language acquisition cannot be overlooked for the vast majority of learners, especially as attaining native-like fluency is judged not only upon one's ability to communicate the mechanics of a language, but also one's savvy in navigating more subtle nuances of the language. Vygotsky (1978) posits, "When faced with such a challenge, the children's emotional use of language increases as well as their efforts to achieve a less automatic, more intelligent solution... Upon being deprived of the opportunity to engage in social speech, children immediately switch over to egocentric speech" (p. 27), which he defines as "the transitional form between external and internal speech." It is the external, or social, speech that leads to further cognitive development and higher-order thinking. Consequently, depriving a student of linguistic output could also deprive him or her of access to proficiency in the language being acquired.

The portion of Krashen's (1998) argument regarding students' anxiety in speaking before their peers bears further consideration. He posits in his *affective filter hypothesis*

(Krashen, 1981), that a pupil's level of anxiety, motivation, and self-confidence can either assist or hinder the pupil's capability to digest the instruction of a new language. Thus, for the language courses to which Krashen's (1998) criticism refers, he condemns the need for students to produce verbal output. However, perhaps the author does not consider or explore pupil-grouping options as a means of allowing for less stressful student output. Cohen (1994) notes, "The group method will provide far more active and relevant practice than having students take turns in making a speech to the whole class" (p. 17). She continues, "If the instructor of the classroom where children need to increase oral proficiency in English sets up a series of tasks that stimulate children to talk to each other, using new vocabulary associated with an interesting task, the possibilities for active language learning can be greatly enhanced" (p. 16). Not only should students be given such opportunities, but also they should be invited to practice their language dexterities on a daily basis (Calderón, 2007). Krashen's omission of this possibility seems contrary to his own "i + 1" theory (Krashen, 1981) – language acquisition occurs when one begins to understand input consisting of nuances just beyond his or her existing aptitude – and its definitive reference to a Vygotskian (1978) idea, the zone of proximal development, for which a social context is mandated.

Barwell (2009) highlights three linguistic tensions that tend to arise specifically in mathematics classrooms: those between mathematics and language in general, between academic language and everyday vernacular, and between the languages that students use at home and school language. For the first tension, research has found that more successful teachers of ELLs tend to be those who address language *and* computation in their mathematics courses. Moreover, Adler (1998) observed that teachers who offered this kind

of instruction in their classes sometimes encountered a “dilemma of transparency” (p. 31), wherein they might spend more time on language than on mathematical concepts. The key is to find balance between the visible, where language is an accessible resource for all students, and the invisible, where language is used as a vehicle to further mathematical understanding.

Often, the assumption is that teaching language in mathematics equates to teaching vocabulary. However, Moschkovich (1999a) argues terminology is not all that should be taught, but also student participation in mathematics dialogue should be supported through strategies such as “modeling consistent norms for discussions, revoicing student contributions, building on what students say and probing what students mean” (p. 18) – what Barwell (2009) calls “broader aspects of language” (p. 7). More discourse-based instruction emerges as the differences between academic language and everyday language appear – Barwell’s second tension. In addition, another of Adler’s (1998) dilemmas also appears – the dilemma of mediation. Within this issue lies the problem of validating students’ ideas and contributions, most likely expressed in everyday vernacular, while pushing students to employ more technical, mathematical terms and to do so correctly. Learning to determine the mode of language to be used is a complex level of language proficiency, and one that even students who are adept in everyday vernacular may have trouble navigating (Gibbons, 2003). No remedy or formula for successfully alleviating this dilemma has been attained at this point; however, Barwell (2009) suggests that this may be an area wherein a teacher’s beliefs play a key role in how he or she navigates through this issue, which is at the heart of either empowering or disempowering a pupil. “Respect, or

disrespect, for community members and their values is conveyed through the lessons teachers create” (Torres-Velasquez & Lobo, 2004/5, p. 250).

Friere (2005) defines dialogue as “the encounter between men, mediated by the world, in order to name the world” (p. 88). He continues, “dialogue cannot occur between those who want to name the world and those who do not wish this naming – between those who deny others the right to speak their word and those whose right to speak has been denied them.” Thus, teachers should encourage the participation and contributions of their students in class discussions in order to enable students to make meaning of the world and, further, in the name of humanization. In contexts wherein there is great social distance, Gibbons (2003) suggests the employment of some kind of mediation in order to assist students in their academic achievement – especially when this disparity involves language and life experiences. Further, the ultimate goal must be creating a climate of collaboration between the teacher and the pupils. For according to Barnett-Clarke and Ramirez (2004), both the teacher and the student must be engaged in both receptive and expressive communication in order for effective discourse to occur.

Chapin and her peers (2009) suggest five tools to assist mathematics instructors with this process: revoicing, asking students to restate others’ comments, asking students to apply their own reasoning to others’, prompting, and wait time. In the revoicing “talk move,” the instructor restates a pupil’s comments, often offered in informal language or with misused academic terms, and asks for the same student’s verification of the statement (Chapin et al., 2009). This procedure allows the student an opportunity to participate in the discussion and clarify his or her own thinking, but also gives space for the instructor to

validate the student's contribution and either conceptually or linguistically correct the child in an unthreatening manner. Gibbons (2003) similarly refers to this move as *recasting*, and adds that in order to facilitate this correction, another move of recontextualizing the student's comments may need to occur wherein the teacher focuses the student explicitly on the registerally appropriate language, including syntactic processing (Swain, 1985).

The move of requesting a different student to restate another one's comments shifts the revoicing move from the teacher to a child's peers. An idea is reiterated, offering more processing time to the class. The teacher obtains evidence that other students are engaged in the dialogue, and finally, the student who offered the idea is validated, as his or her peers have listened to and considered the student's viewpoint.

Much care has to be taken when asking students' classmates to apply their own reasoning to another student's idea (Chapin et al., 2009). The teacher cannot assume any bias in the discussion, managing not only what he or she says and the tone in which statements are made, but also his or her nonverbal body language. In addition, students cannot simply agree or disagree with a remark, but must offer a full explanation of their positions. The prompting move then furthers the class dialogue, requesting other annotations. Gibbons (2003) adds that this move might also include questioning the student who offered the original idea in order that he or she might self-correct the idea being discussed. The author also suggests allowing this same student to restate or *reformulate* the idea being expressed using appropriate questioning techniques. For English-language learners this process would allow the student to further develop his or

her ability to negotiate meaning (Swain, 1985), advancing the student's progress toward achieving native-like proficiency in English and in the register of mathematics.

Finally, in soliciting pupils' input, wait time may be required. For this move, more self-control is required of the instructor in that he or she must become comfortable with silence and perform this act consistently in order to master it. Saphier and Gower (1997) noticed in an instance of this practice that "many students who ordinarily did not answer did so, the answers tended to be in full sentences rather than single words or phrases, the answers were at a higher level of thinking, and students were more likely to start responding to each other and to comment on each other's answers" (p. 309). Thus, this one talk move provides room for the other moves for effective discussion, resulting in more comprehensible output (Swain, 1985). It should also be noted that in more diverse classrooms, the "students who ordinarily did not answer" tend to be those who are English-language learners (Chapin et al., 2009), who are of historically underserved racial or socioeconomic backgrounds, or possess learning disabilities. Therefore, wait time provides more equal access to classroom achievement for all.

Barwell's (2009) third tension – the conflict between students' home languages and official school languages – speaks to the aforementioned political nature of schooling. Setati (2005) noted this tension in classroom discourse observed in South Africa, wherein students' home language was used for more conceptual and contextual explanations, while English, the "international language" (p. 459), was used for procedural discussions as well as all assessments. The teacher of the class observed in Setati's study was of the same cultural and linguistical background of the students, and so she was able to codeswitch for

the learning benefit of her children. However, while she appreciated the policies that allowed her to use the students' home language, she was also concerned about the students' English proficiency beyond the school. In Pakistan, this friction appeared as a result of the 2005-2006 curriculum reform initiatives which instituted the use of English for instruction in the content areas of mathematics, science, and computers (Halai, 2009).

“... so many years we have been made to feel inferior with our languages. Because, you know, more often than not in meetings, everywhere, you couldn't just stand up and express yourself in vernacular. I mean, you would be ridiculed as somebody who cannot express themselves” (p. 459).

Nevertheless, Beykont (2002) purports success for linguistically diverse students resides in the acknowledgement of the power of both the mainstream and home cultures, and thus, both the mainstream and home languages.

This tension is encompassed by Adler's (1998) third dilemma as well, wherein “decisions in the classroom often revolve around the tensions between developing pupils' English (the language of instruction) and ensuring pupils understand the mathematics” (p. 26). Further, as the teacher tries to model the school language, a sub-dilemma of the teacher possibly speaking too much emerges. There is no remedy to these quandaries; at best, an instructor can be reflective and acknowledge the existence of these dilemmas, and then respond to the needs of their students with as much care and equity as possible. Halai (2009) suggests that introspection and examination of one's hidden curriculum is especially necessary where immigrants are involved since “there is positioning of the immigrant context as ‘different’ from the common or normal context familiar to the

teacher” (p. 50). The author continues that issues of power and status emerge from this labeling, as “the immigrant context is subordinated in comparison to the dominant context,” giving the student the impression that his or her home language is incapable of contributing to his or her learning experience. As the number of students in U.S. schools who originate from “non-dominant cultural and linguistic backgrounds [and] teachers remain overwhelmingly White and, presumably, monolingual speakers of English” (Bunch, 2010, p. 351), the issues of status, power, and social justice must be approached as well.

Considering Social Justice

By the end of the 1920s in the United States, the industrial age had fully materialized (Kliebard, 2004). With this shift in the country’s economic resources came unrest about the social system of America in that, up until this time, it seemed to only benefit the more privileged of the population rather than all citizens. Counts (1997) argued, “If the machine is to serve all, and serve all equally, it cannot be the property of the few” (p. 29). Within this unrest emerged discontent regarding the education being offered in public schools and how the new influx of students would be prepared to enter and contribute to this evolving society. By the 1930s, Counts and other educational pioneers began promoting the notion of *social reconstructivism* in school curriculum, wherein curriculum was viewed as “the vehicle by which social injustice would be redressed and the evils of capitalism corrected” (Kliebard, 2004, p. 154). Consequently, what would be the role of the teacher? Counts (1997) suggested that teachers “must bridge the gap between school and society” (p. 25); thus, “To the extent that they are permitted to fashion the curriculum and the procedures of the school they will definitely and positively influence the social attitudes, ideals, and behavior of the coming generation” (p. 24). It can, therefore, be concluded that, at least

within the social reconstructionist paradigm of curriculum theory, teaching is inherently a political activity (Bartolomé, 2002; Cochran-Smith, 2006; Torres-Velasquez & Lobo, 2004/5). Further, “in societies characterized by unequal power relations among groups, pedagogy is never neutral” (Cummins, 1997, p. 105).

When considering the diversity of students present in American public schools, the political nature of teaching evolves to include the valuation of the nationalities, cultural backgrounds, and learning styles represented in one’s classroom – once deemed by Horace Kallen (a former pupil and disciple of John Dewey) as *cultural pluralism* (Glazer, 1997). In the 1840s this concept was at the center of educational debates as Catholic leaders fought for equal treatment of Catholic students and the inclusion of the Catholic Douay translation of the bible. In the 1880s, cultural pluralism took on another face in the struggle of German immigrants of the Midwest region of the United States who were seeking permission for their children to be taught in German. The reign and propaganda of Hitler during World War II prompted the resurgence of cultural pluralism. “It was in the interest of the war effort to teach equality and tolerance” (p. 276) as opposed to Hitler’s message of hatred. However, this revival was short-lived as the 1950-60s brought about a nationalist movement due to Russia’s release of Sputnik and the desegregation movement.

Desegregation of the 60s brought about a lingering paradox (Glazer, 1997). While civil rights leaders were fighting for equality in public schools, that equality was accompanied by assimilation. For “If whites’ education had precious little of cultural pluralism or multiculturalism in it, why should that be changed for blacks?” (p. 276). The resulting climate in many schools was and continues to be a condition Irvine (1991) refers to as the *lack of cultural sync*. The author notes, “Because the culture of black children is

different and often misunderstood, ignored, or discounted, black students are likely to experience cultural discontinuity in schools, particularly schools in which the majority, or Eurocentric persons, control, administer, and reach" (p. xix). Delpit (2006) adds that these students of color may also be deemed unready for the academic challenges that are offered to their peers simply due to the lack of linguistic parallelism to their teachers. Some of the consequences of the climate created by these misgivings are low expectations exuded by teachers, personality and cultural clashes between students and instructors, and the ostracism of students who seemingly behave more Eurocentrically than their peers, resulting in academic failure and educational disinterest for these students.

Gay (2000) cites several factors that influence teachers' expectations for their students, as well as the classroom opportunities designed for them – racial identity, gender, ethnicity, physical appearance, culture, social class, and home language. Regarding ELLs, Fillmore and Snow (2002) warn, "Too few teachers share or know about their students' cultural and linguistic backgrounds or understand the challenges inherent in learning to speak and read Standard English" (p. 7). Krashen (1982) believes that an *affective filter* develops as a result of such cultural disparity, creating an anxiety within these students that not only distracts them from learning, but also causes them to withdraw socially. Cummins (1997) adds, "For subordinated group pupils, the price of admission into the teaching-learning relationship... is frequently renunciation of self" (p. 108). Despite all of these dynamics in the classroom among students and teachers, somehow students are still expected to achieve academic excellence (Ladson-Billings, 1995) and their inability to reach this goal will have serious consequences for their futures in such a progressively more technological society (Gutiérrez, 2002).

Cummins (1997) purports that the relationships between teachers and students “always either reinforce coercive relations of power or promote collaborative relations of power” (p. 105). Thus, in the effort to culturally synchronize with their pupils, teachers must consider their beliefs about race and culture compared to those of society at large, and then make deliberate considerations as to how to approach the students in their classrooms. One perception that should be addressed is that the racial or cultural classification of students into a particular group does not equate to them all having the same cultural experiences (Ladson-Billings, 1995). In reference to Latino students, but generalizable to all students, Moschkovich (1999b) warns, “Although these students will have some shared experiences, such as some relationship to the use of Spanish, there will also be many differences among these students’ experiences, either at home or in school” (p. 10). Lucas and colleagues (2008) add, “They enter U.S. schools with varying levels of oral proficiency and literacy (in both English and their native language) as well as prior knowledge of and experiences with subject matter” (p. 366). Therefore, teachers should get to know their students and avoid deficit models when teaching pupils of varied backgrounds in order to assist them in their quests for academic achievement, more specifically in mathematics (Brisk et al., 2002; Moschkovich, 1999b). In addition, Lucas and her peers (2008) promote “safe, welcoming classroom environment[s] with minimal anxiety about performing in a second language” as essential conditions for ELLs’ learning. Pedagogical practices of this nature address the climate of the classroom and the school, affecting pupils’ motivation to learn and, consequently, their ability to receive instruction.

Steele (1997) offers that there is an additional climatic phenomenon of which faculty should be aware, especially for students of color and women, called *stereotype*

threat. He defines this occurrence as “the threat of being viewed through the lens of a negative stereotype or the fear of doing something that would inadvertently confirm that stereotype” (Steele, 1999, p. 3). Such a hindrance anyone can experience as people can be typecast for numerous reasons – e.g. gender, age, sexual orientation, race, or language. However, within academia, more specifically mathematics and the sciences, women and minorities tend to experience this kind of intimidation.

There are two aspects of stereotype threat of most concern. The first is that it is not usually tangible, but rather a part of the hidden curriculum of a school or classroom – the routines and expectations not explicitly taught but that are prevalent in an academic setting. This is often what produces Irvine’s (1991) lack of cultural synchronization and causes the student experiencing the threat to become mistrustful and question the teacher’s and fellow students’ motives. “Why was that old roommate unfriendly to him? Did that young white woman who has been so nice to him in class not return his phone call because she’s afraid he’ll ask her for a date? Is it because of his race or something else about him” (Steele, 1999, p. 3). From the perspective of preservice teachers, such a threat may be heightened due to their typical view that the American social order is fair and just, causing them to resent exploring theories that “challenge some of the dominant-culture ideologies they unconsciously hold” (Bartolomé, 2002, p. 169).

The second aspect of stereotype threat of concern in this study is that it mostly affects students who have skill in a particular subject and have bought into the importance of schooling (Steele, 1997). This particularly relates to immigrant students who were successful in their native countries but are often remediated when beginning school in the U.S. Some children are placed in special needs classes erroneously, while others are

ostracized by fellow students or ignored by their classroom teachers. Stereotype threat in these cases often leads to disidentification by the student, wherein the student begins to withdraw and “pain is lessened by ceasing to identify with the part of life in which the pain occurs” (Steele, 1999, p. 3). This withdrawal can lead to the student no longer caring about performing well in the class or in school, in general, which, in turn, leads to the decrease in the student’s motivation and poor school achievement, and possibly even to an *inferiority anxiety* (Steele, 1997).

Combating stereotype threat is not always easy, but is definitely necessary. Steele (1997) suggests creating optimistic student-teacher reports, giving students more challenging work rather than remediation, and emphasizing the expandability of one’s intelligence given experience and guidance. In addition, the author promotes affirming students’ belonging in a class or school, providing role models from the stereotype-threatened group, and valuing multiple perspectives and approaches. The latter point is one that is growing in acceptance in mathematics and has initiated the redefining of the mathematics classroom culture as “the *product* of what the teacher *and* pupils bring to it in terms of knowledge, beliefs, and values, and how these affect the social interactions within that context” (Nickson, 1992, p. 111, italics added). It is this type of climate that is conducive to the critical dialogue necessary for the existence of culturally relevant pedagogy (Ladson-Billings, 1995), as well as the student engagement and output vital for the manifestation of LRP (Lucas et al., 2008).

Ladson-Billings’ (1995) theoretical model of *culturally relevant pedagogy* can be defined as an instructional practice, or collection of practices, that “not only addresses student achievement but also helps students to accept and affirm their cultural identity

while developing critical perspectives that challenge inequities that schools (and other institutions) perpetuate” (p. 469), engaging students in a form of praxis (Gebhard et al., 2002). While LRP does not engage in the critical dialogue associated with culturally relevant pedagogy, the validating nature of the latter is crucial for engaging students in successful language instruction. To begin the process of employing culturally relevant pedagogy, the instructor’s conception of his or herself and of others, namely the students, must be confronted (Ladson-Billings, 1995). Characteristics that the researcher found among culturally relevant teachers were:

- The belief that “all students were capable of academic success” (p. 478)
- The notion of pedagogy as an art – “unpredictable, always in the process of becoming”
- The view of themselves as “members of the community”
- The philosophy of teaching being a way to contribute to the community
- The notion of teaching as a mining, problem-posing process as opposed to a “banking” or depository practice (Freire, 2005)

It is through these aspects of their belief systems that the instructors in this study were able to form positive relationships with their students and improve their pupils’ acquisition of knowledge (Ladson-Billings, 1995). Transitioning from belief to action, Cummins (1997) advocates for teachers to encourage their students’ linguistic diversity, to forge relationships with parents and communities that are culturally diverse, and to seek out and utilize pedagogy that enables students to “resist devaluation and ‘take control of their own lives’” (p. 109). Gay (2000) also deems teachers’ facilitation of community within the classroom, as well as their connectedness to students’ home communities, as vital to

providing educative experiences that are validating, empowering, and emancipatory, allowing for pupils' intellectual transformations rather than assimilations.

While many educators and education researchers may support the premise of culturally relevant pedagogy, there are naysayers who would argue that embracing cultural diversity may subvert a country's national identity (Cummins, 1997; Glazer, 1997).

Opponents of bilingual education often argue that the inclusion of more than one language in mainstream classrooms may "invariably problematise the culture, attitudes and language use of subordinated group... when subordinated groups refuse to accept their preordained status and demand 'rights'" (Cummins, 1997, p. 107). Other critics suggest:

"... large sections of the American population, particularly poor racial and ethnic groups that have been subjected to discrimination, will receive an education that attributed blame for their condition to the white or European majority and thereby worsen political and social splits along racial and ethnic lines" (Glazer, 1997, p. 274).

At the heart of these types of critiques is two types of fear – fear of the loss of power by a ruling group and fear of the unrest and uprising of those who have been historically oppressed by this group.

Freire (2005) proposes that in order for the oppressed and the oppressors to be liberated, there must be a struggle that ensues from the oppressed, while Delpit (2006) assigns the majority of the responsibility to the oppressor. The author recommends that the unwritten rules of those with power be explicitly taught to those without power in order to allow the latter to participate in the existing "culture of power" (p. 24), positioning teachers as "cultural interpreters" (Beykont, 2002, p. xxx) and "change agents to create

more just and democratic schools and learning opportunities” (Bartolomé, 2002, p. 182). Delpit (2006) adds, “If such explicitness is not provided to students, what it feels like to people who are old enough to judge is there are secrets being kept, that time is being wasted, that the teacher is abdicating his or her duty to teach” (p. 31). On the contrary, Freire (2005) contends that instigation by the ruling class often translates as a “false generosity” (p. 44), continuously producing an “unjust social order.” However, the author admits, “The peasant feels inferior to the boss because the boss seems to be the only one who knows things and is able to run things” (p. 63), justifying the need for the powerless to be instructed in the rules of the powerful.

In either approach, the oppression of inequity must be overtly challenged. After all, “to act as if power does not exist is to ensure that the power status quo remains the same” (Delpit, 2006, p. 39). It is this behavior in educators that propagates the existing societal inequities, as the instructors refuse to “identify and interrogate their negative, racist, and ideological orientations” (Bartolomé, 2002). To combat the status quo, Cummins (1997) advocates for the collaboration of all educative participants for the purpose of generating an additive power relationship wherein “each [pupil] is more affirmed in her or his identity and has a greater sense of efficacy to create change in his or her life or social situation” (p. 111). Delpit (2006) continues that “education, at its best, hones and develops the knowledge and skills each student already possesses, while at the same time adding new knowledge and skills to that base” (p. 67-68) – Vygotsky’s (1978) “zone of proximal development” concept. For linguistically diverse students, this ideology results in teachers approaching these pupils not as children with “poor” English-language competence, but

rather, as “individuals with another language an cultural experience who have varying degrees of English proficiency” (Brisk et al., 2002, p. 114).

Empirical Investigations

According to Gutiérrez (2002), “when teachers do not understand the relationship between language and mathematics instruction, they tend to hold either unreasonably high expectations of English language learners without the necessary support structures or unreasonably low expectations, such that students are denied access to high-quality mathematics” (p. 1051). Considering this view, this section will present various empirical studies that contribute some insights into the merger of language and mathematics teaching and learning.

Cahmann and Remillard (2002) followed two elementary teachers who taught in low-SES, multicultural schools to assess the ways in which the teachers embraced reform-based mathematics curricula with diverse populations. The Puerto Rican instructor, Linda Arieto, focused the instruction of her bilingual class on creating a bridge between home and school and on drawing on her own experience as an ELL. Zoey Kitcher, the Caucasian teacher, concentrated more on the open-classroom model that her school had adopted. For this model, the teachers developed an integrated curriculum based on school themes and everyone used manipulatives. Needless to say, the researchers found multiple differences in the culture and instruction in these two classrooms.

Because of Linda’s shared culture and background with many of her students, she was able to “empathize with their life experiences and ... provide a nurturing transition from home to school” (Cahmann & Remillard, 2002, p. 188). She promoted habits of mind in her classroom such as self-control and respect for authority, and used her fluency in

Spanish to instruct her students in both Spanish and English. However, Linda struggled with translating the mathematics-specific terminology in the text into Spanish, as her own tutelage in mathematics had been completely in English. She also had limited resources as far as translated materials, and she encountered additional pressure to have her students ready for mainstream classes by the second semester of the school year.

Zoey, on the other hand, opted for a “color-blind” approach and chose not to address class or culture explicitly in her instruction. However, her own comments about her class hint at the hidden curriculum that often ostracizes students of color, leading to lack of cultural synchronization (Irvine, 1991) and to a culture conducive to stereotype threat (Steele, 1997):

“A child who’s more open to being, um, critiqued – who understands what to do in the classroom – is easier for me to deal with than the child who, um, walks in every day like, what’s going on here, who acts like he’s, you know, she’s a stranger to the classroom” (Cahmann & Remillard, 2002, p. 190).

In addition, both Zoey and Linda had difficulty with mathematics themselves as students and thus, found it difficult to teach in a way that would make the subject more interesting than fear-laden. Despite this, Linda included overlooked units in her instruction such as statistics, although she did not stress accuracy and validity. Zoey grew in her mathematics knowledge to begin utilizing another teaching resource rather than the assigned textbook and also to emphasize conceptual understanding as opposed to correct answers.

Cahmann and Remillard (2002) conclude that reform-oriented mathematics seems to require a deeper knowledge of mathematics and more preparation time than elementary teachers typically possess. However, teachers do need to build trust with their students by

making cultural connections with them in addition to supplying authentic mathematical experiences that focus on conceptual understanding. It is the synergy of both of these aspects of mathematics instruction that provides the optimal condition for all students to learn the subject.

Another case study was conducted by Gebhard, Hafner, and Wright (2004), wherein an ELL's mathematics experience in Wright's third-grade class was followed throughout the school year. Wright was a student in Gebhard's two-semester course regarding second-language acquisition and academic literacy development, and, thus solicited assistance of Gebhard as the expert and Hafner as the unbiased observer in order to adjust her mathematics lessons to address the needs of her ELLs. More specifically, Wright was interested in learning to teach her students how to answer open-ended problems in the curriculum as well as on high-stakes tests, as "almost all of the ELLs in her school had either skipped this open-ended sections of the fourth-grade math MCAS or thought it was a multiple-choice question and responded accordingly" (p. 39).

To begin the process, Gebhard instructed Hafner and Wright in inventorying and categorizing the various ways in which reading and writing were being required in mathematics. The team then developed descriptions of the linguistic elements of the curriculum – vocabulary, grammatical constructions, phrase, and organizational schemes. In addition, Wright prepared her class to begin tackling the language demands of mathematics by referring to them as a game – one they could win, if they know the rules. From the data analysis, Wright made changes to her mathematical pedagogy:

- She grouped students heterogeneously by language and mathematical ability in order for students to be able to learn from one another.

- She taught students explicitly how to function in groups – wait time, equal time in conversations, rephrasing, and how to deal with conceptual conflict.
- She had students discuss and create visual representations of their thoughts before starting to write their responses to problems.
- She modeled different ways of beginning a problem and taught students how to identify and respond to directives such as *label*, *describe*, and *give evidence*.
- She created a word wall not only for key vocabulary, but also for “words and phrases that signaled different kinds of operations” (p. 40).
- She used the writing and revising process that students learned during the language-arts portion of the class to have students draft and revise their mathematics responses. Within this process, students 1) shared their work with peers using guides, graphic organizers, and worksheets; 2) made revisions based on this feedback and based on the feedback they received in conferences with the teacher; and 3) proofread and revised their work before final submissions.

For the purposes of the study, one ELL was selected for closer monitoring of her progress throughout the school year. The student, Marisol, initially wrote computations and solutions without demonstrating any strategies and without indicating the numbers that she used to obtain her solutions. She attempted to write the required verbal descriptions but had great difficulty with word choices, making her writing difficult to comprehend to an outsider of the class. By the middle of the school year, Marisol was able to generate graphic organizers that showed how she was thinking about problems. Her word selections were much more appropriate and she even used sequencing words such as *first* and *then*. By the end of the school year, Marisol was able to create her own word

problem and demonstrate how to solve it correctly. She even approached Wright for more work, although she had proclaimed that she hated mathematics at the beginning of the school year.

At the secondary level, Gutiérrez (2002) followed three mathematics instructors with a proven record of successful ELL instruction, i.e. these teachers were renowned for promoting Latino students into calculus classes and the students were able to perform as well or better than their monolingual peers. Thus, the author and her staff observed these teachers for approximately a year and a half to determine attributes of their practice that can be implemented across the field to assist in the instruction of ELLs. It should be noted that two of the teachers were Caucasian and one was Latino, the ethnicity of the majority of the ELLs at this school. Interestingly, both of the Caucasian instructors had been active in the Civil Rights Movement of the 1960s and expressed ongoing commitment to social justice and high-quality education for all students.

Across all three practices, there were some common strategies employed consciously and unconsciously that contributed to the success of their students (Gutiérrez, 2002). Each of the teachers made the effort to get to know their students personally as well as academically. “On a regular basis, the teachers could be found joking with students, bringing in food to share (even though it was against school policy to eat in classrooms), sharing their personal lives with students, and showing interest in the students’ personal lives” (p. 1074). In addition, each teacher knew the linguistic abilities of his students – who was monolingual, fully bilingual in Spanish and English, or bilingual but Spanish-dominant. At no time were the instructors observed overgeneralizing about their students’

mathematical or language capabilities, but rather, they understood their students individually.

The three teachers all employed cooperative learning or some other kind of peer interaction routinely in their instruction (Gutiérrez, 2002). Students were regrouped every 3-4 weeks, and the instructors were deliberate in making sure there were fully bilingual students in groups where there were students who needed assistance with English. Pupils were allowed to speak in their first languages for peer interactions even if there were monolingual students in the group and despite all presentations to the class having to be in English. The teachers “relinquished their authority and trusted the students” (p. 1075) – a skill that tends to be difficult for many mathematics instructors – and “made sure that students worked with each other across superficial boundaries to develop alliances.” This practice allowed students to take ownership of their own learning and feel more investment in the classes, as “When students can identify with the school and with the teachers, they are more likely to feel that they have a place in the classroom as learners” (Torres-Velasquez & Lobo, 2004/5, p. 251). Also, while some teachers may shy away from this amount of communication in their classrooms where there exists a larger number of limited-English speakers, these three instructors welcomed the challenge. Cohen (1994) notes that for middle and secondary students, “Unless the students can communicate scientific ideas, analysis of a social problem, or the logic behind a deduction in mathematics, they will have difficulty with advanced coursework in these subjects” (p. 47). This further supports the track record of these teachers in their propelling ELLs into higher-level mathematics.

Another practice of note is that the three instructors examined in this study were not constrained instructionally to their textbooks, but rather, they viewed their books as resources and not the curriculum itself (Gutiérrez, 2002). About 80% of the materials utilized for assignments and activities were created by the teachers themselves and reflected more closely the experiences students were encountering in the classroom. The textbooks were used more so “to familiarize students with the kinds of language and representations of mathematics problems that they might encounter in college-level courses or on standardized tests, which held significant life consequences for students” (p. 1077). As Delpit (2006) encourages, “we must take the responsibility to *teach*, to provide for students who do not already possess them, the additional codes of power” (p. 40). By developing their own materials and using the textbook in this manner, the teachers established themselves as mediators, bridging of the gap between students’ external languages and the language of mathematical academia, and “critically assessing taken-for-granted assumptions about the nature of language, learning, and diversity” (Gebhard et al., 2002, p. 224).

These teachers were not formally trained in ways and methods to address the needs of ELLs, but were flexible enough to try various approaches to reach their students (Gutiérrez, 2002). However, how much more effective could these instructors have been had they accessed any existing research on instructing ELLs? Thus, Gutiérrez recommends adjustments at the teacher-education level. The author posits, “Exposing preservice teachers to the literature on effective learning environments for language minority students and Latina/o students is another step in the right direction, especially for secondary school teachers whose programs tend to emphasize mastery of subject matter,

not development of the learner” (p. 1082). The significance of social context in learning should also be promoted, and preservice teachers should be given the opportunity of utilizing the guidance of research to even develop their own strategies. Mathematics was once categorized as *culture-free* or *universal*; however, research such as this study is increasingly showing that sensitivity to the culture of one’s pupils is crucial to the success of all students and, thus, simply good practice.

Regarding mediation and the revision of language in mathematics, a study was conducted that analyzed both the qualitative and quantitative impacts of modifying the language on a standardized test, in particular NAEP (Abedi & Lord, 2001). Such modifications included changing unfamiliar non-mathematics terminology to more familiar terms, passive voice verbs to active voice, complex questioning to simpler phrasing, and abstract references to more concrete. A sample of eighth-grade students ($n = 19$) of various SES and academic and language ability was shown four sample items from NAEP and four linguistically revised versions of those same questions. None of the mathematics-specific terminology was changed, and experts in mathematics were consulted to assure that meaning of the problems would not be lost in the revised versions. When these students were interviewed regarding their interpretations of the original and altered questions, 63% of the students preferred the modifications. The study was repeated with a second set of four questions and revised versions, and about 83% of the pupils chose the modified versions. Students remarked, “It seems simpler; you get a clear idea of what they want you to do... It’s easier to read, and it gets to the point so you won’t have to waste time” (p. 222).

For the quantitative phase of the study, over 1,000 students were given two versions of a test booklet – one with ten sample NAEP questions, ten revised questions, and

then five sample questions deemed as consisting of language that was not challenging (Abedi & Lord, 2001). The second booklet had the converse questions for the first twenty problems and the same five linguistically simple questions. Three key findings resulted. Most of the children, particularly the ELLs, scored better on the modified questions. Low SES students had a higher percent gain in their scores on the altered questions than those of high SES, and those enrolled in low- and average-level mathematics classes had a higher percent gain than student in higher levels as well. The latter two findings were not statistically significant, but worth considering for pedagogical purposes in addition to the first result. In one instance of the qualitative portion of the research, a student “substituted an active verb... replacing a less familiar construction with a more familiar one” (p. 223), indicating an instructional influence. Non-academic sources can be altered or have alternative versions; however, teachers who are well-apprieved of the language demands of mathematics can make a large impact by teaching their students – ELLs and low SES, in particular – how to decode the language and revise it for themselves. This would equip students with the tools they need to encounter mathematics questions at any level and to overcome linguistic challenges that typically hinder their success, converting “limit-situations” (Freire, 2005, p. 99) to liberating ones.

Interpretive Summary

According to Adler (1998), teaching mathematics to all students involves “enabling *epistemic access* for all to *appropriate mathematical knowledge* in school, and enabling the *participation and inclusion* of diverse voices in the mathematics curriculum” (p. 24). When considering the incorporation of ELLs into the secondary mathematics classroom, access to the content and the language has to be addressed. As one of Dong’s (2004) graduate

students adequately assessed, “English language learners [do] not have the luxury of time to develop basic English skills before learning the subject matter” (p. 204), as they are battling to “catch up with their peers and graduate.” Thus, as “A critical role of the public school is to assure that all students, regardless of home language, master the language of the mainstream” (Feldman, 2002, p. 114), the role of the mainstream mathematics instructor must expand to include the ability to embrace and account for both cultural and linguistic diversity, varying learning styles and abilities, and also the content. Such a stance calls for the transition of teachers’ beliefs to students’ differences as resources, rather than issues (Gebhard et al., 2002).

Unfortunately, the secondary mathematics instructor is not privy to much assistance from research. Most literature regarding the instruction of ELLs is geared to humanities subject areas, ELL specialists, or elementary teachers, or is purely theoretical. The literature cited in this review makes the case for the need for linguistically diverse classrooms on all levels and offer some pedagogy strategies for creating a classroom culture conducive for ELLs’ learning. However, this research does not incorporate explicit methods of teaching the language of mathematics, nor teaching the mathematics register in the upper grades. Thus, empirical research regarding effective practices for teaching ELLs in mathematics and the preparation of secondary teachers to do so can make an important contribution to the field, which is the goal of this dissertation study.

Dong (2004) suggests that there are four major areas for the instruction of ELLs in which secondary preservice teachers should be developed:

- Building empathy toward second language learners’ difficulties and cultural differences

- Increasing understanding of the process of second language acquisition
- Adapting the curriculum and instruction to these students' cultural and language needs, and
- Integrating discipline-specific language and literacy skills into the arena of instruction

The changes that a teacher educator instituted in the secondary mathematics methods course during this study incorporated these areas. Thus, this mixed-methods study examined the effects of the content-and-language integration-related alterations implemented the fall semesters of 2009 and 2010. Chapter Three outlines the methodology that guided the data collection and analysis for the study.

Chapter 3

METHODOLOGY

This chapter describes the design of the research, a description of the participants, the contexts of the study, the methods of data collection (including the instruments utilized), the modes of data analysis, and the limitations of the methodology. In addition, the chapter will reiterate the purpose and research questions for the study, and include the rationale for the mixed-method approach as the data collection is discussed.

The purpose of this study was to explore the effectiveness of an instructional intervention administered during a required secondary mathematics methods course in preparing undergraduate and graduate-level preservice secondary mathematics teachers to address the linguistic needs of ELLs. The questions guiding this investigation were:

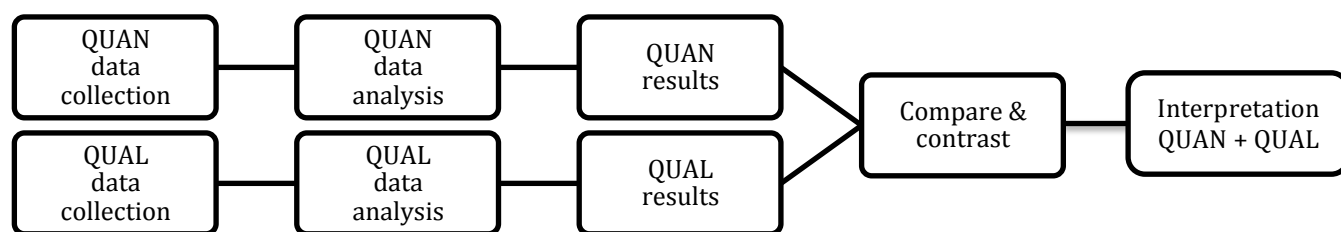
How does a content-and-language integration intervention effect the preparation of secondary mathematics preservice teachers in order to improve instruction for diverse learners, more specifically ELLs, in mainstream mathematics classes?

- a. Do the preservice teachers' beliefs change regarding their ability to provide content and language-integrated learning opportunities for ELLs in mathematics? If so, how?
- b. Do the preservice teachers' ability to recognize academic-language challenges for ELLs evolve during the secondary mathematics methods course? If so, how?

Research Design

To answer the research questions, a mixed-methods approach was employed as both quantitative and qualitative data were collected for analysis in this study. More specifically, the convergence model of triangulation design typical for mixed-methods research was employed (see Figure 3.1), wherein both quantitative and qualitative data were used to determine the effects of the intervention (Creswell & Plano-Clark, 2007). Through expanding the study from quantitative means into more qualitative approaches, the researcher is able “to explore the phenomena as they naturally occur, thus allowing [the researcher] to organize and describe the phenomena with depth and richness” (Hill et al., 1997, p. 518). Greene and Caracelli (2003) posit, “mixed methods inquiry can be a means for exploring differences; a forum for dialogue; or an opportunity to better understand different ways of seeing, knowing, and valuing” (p. 107). Thus, applying the authors’ viewpoint, this study explored the different ways that a content-and-language intervention in a mathematics-methods course was received by preservice teachers in the course and analyzes evidence of how the teachers see, know, and value the mathematics register.

Figure 3.1. *Convergence Model of Triangulation Design for Mixed-Methods Research (Creswell & Plano-Clark, 2007)*



A closed-question, quantitative survey with a qualitative drawing component was conducted at the beginning of each fall semester and then readministered at the end of each semester with the addition of a few open-ended questions in order to measure the impact of the academic-language intervention. To corroborate the results of the pre- and post-surveys, pre- and post-interviews were conducted with a purposeful sample of the course participants. In addition, course and pre-practicum artifacts were collected to further substantiate the survey results. This multi-layered approach would be deemed by Creswell and Plano Clark (2007) as the *triangulation design* of mixed-methods research, wherein “complementary findings [are obtained] that strengthen research results and contribute to theory and knowledge development” (Morse, 1991, p. 122). In addition, Lather (1986) warns, “The researcher must consciously utilize designs that allow counterpatterns as well as convergence [in the process of triangulation] if data are to be credible” (p. 270).

Inquiry traditions

The case study was the primary method of inquiry for the qualitative portion of this study. More specifically, as several cases in the form of the five interviewed participants from the fall semester were examined, this study employed a *collective case study* (Stake, 1995) or *multiple case study* (Yin, 1994). Creswell (1998) defines the case-study approach as “an exploration of a ‘bounded system’ or a case (or multiple cases) over time through detailed, in-depth data collections involving multiple sources of information” (p. 61). The “bounded system” is a situation of a particular time and place, which for this study was the secondary mathematics methods course, offered the fall semester of 2010. The multiple sources of information required for this mode of inquiry include Yin’s six recommended

sources of case-study evidence: “documentation, archival records, interviews, direct observations participant-observation, and physical artifacts” (p. 79). The author further purports that there are five applications for the case-study method of inquiry: to explain causal links, to describe an intervention, to illustrate topics within an evaluation, to explore situations where there are no definite outcomes, and to serve as a metaevaluation. This study adopted the more exploratory use of inquiry, as the results of the content-and-language intervention are unpredictable; thus, requiring the examination of the participants’ views and course products in order to assess the intervention’s effectiveness.

Case studies have been faulted for lacking rigor and for their lack of generalizability (Hamel et al., 1993). However, this research was designed to offer for consideration an approach to preparing future secondary mathematics teachers to handle the growing language challenges in U.S. schools, and thus, does not claim that the design of the intervention to be completely universal. Hamel and his peers contend, “The case study has proven to be in complete harmony with the three words that characterize any qualitative method: describing, understanding, and explaining” (p. 39). It is the strength acquired through the detailed description, complexity of design, and use of multiple sources within this dissertation that future applicability will be possible (Rossman & Rallis, 2003).

Access and Entry

Verbal permission was granted by the mathematics course instructor to allow the study to occur in her course each year. Documented permission to conduct research with human subjects was then obtained through the university’s Institutional Review Board (IRB). Written permission was gained from as many preservice teachers enrolled in the course as possible at the beginning of each fall semester. This research is supported by a

grant from the U.S. Department of Education's Office of English Language Acquisition (#T195N070133-08).

Context and Participants

Context

This research study occurred at a private catholic university in the northeastern region of the United States as a part of the larger Content Academic Language (CAL) study, and more specifically, in a secondary mathematics methods course, wherein preservice teachers were instructed in middle- and high-school mathematics pedagogy and research trends. The course is required for secondary mathematics-education majors, both undergraduate and graduate levels, as well as those minoring in middle-school mathematics education. All teacher-education courses at this institution are built around five themes: social justice; knowledge construction; inquiry into theory, beliefs, and practice; diversity, and community collaboration. The methods course was designed to incorporate these themes, as well as to examine and understand the nature of mathematics curriculum as “a multifaceted, human endeavor by which people construct concepts, discover relationships, invent algorithms and models, organize and communicate their thoughts, and address real world problems” (course syllabus) and to self-reflect on one’s beliefs about mathematics teaching and learning.

The course met on Monday afternoons at 4:30 until 6:50 and lasts about fourteen weeks, both fall semesters. Participants were expected to complete weekly course readings as well as attend class weekly. In addition, the preservice teachers were expected to complete pre-practica in local schools while enrolled in the course. The majority of the preservice teachers enrolled in the methods course were either in their first pre-practica

semesters (graduate students) or the second (undergraduates). Those who were not assigned to schools were typically certified practicing teachers or international students, but the number of course participants with such status are very few. For both the fall 2009 and 2010 semesters, three preservice teachers in each course fell into this latter category.

There were seven assignments for the course each semester; the first being a critical analysis of an article regarding current issues in mathematics education. The second assignment was the examination of a case study for which the course participants analyze the study according to the mathematical theme of the case, the pedagogy and context presented, and the evidence of the student learning. Further, the assignment additionally required the preservice teachers to explore and describe their own philosophies about the teaching and learning of mathematics. Another course assignment, the Lesson Organizer/Reflection, required the planning and implementation of a mathematics lesson in an authentic environment, as well as a self-reflective essay regarding what was observed and learned throughout the instructional process.

In response to evolving societal demands, there were four assignments requiring the course participants to engage with current technology – the Newsletter, Website, Geometer’s Sketchpad, and Concept Presentation assignments. The Newsletter assignment required the preservice teachers to test and analyze mathematics software, as well as existing mathematics-related websites. The Website assignment allotted the opportunity for each course participant to develop an individual website as an informational and instructional resource for their own classes. The Geometer’s Sketchpad assignment required the preservice teachers to create instructional applets that can be uploaded to their websites and utilized by their current or future students. Finally, the Concept

Presentation assignment served as the culminating product for the course, wherein small groups of the course participants designed and implemented mathematics lessons, providing advice about the techniques and strategies for concepts presented and integrating technological tools such as Microsoft's PowerPoint program and the SMART Board.

Participants

Preservice teachers enrolled in the secondary mathematics methods classes of the fall 2010 semesters were asked to participate in the study. In 2009, sixteen students were in the course and all agreed to participate in the research. Fourteen preservice teachers registered for the class in 2010; however, all but one of them volunteered to participate in the study. The students' agreement to participate in the study provided access to their pre- and post-surveys, classwork, comments in class, course assignments, and practicum documents for the term.

For the case study portion of the research, "purposeful" sampling (Creswell, 2003; Gall et al., 2010; Maxwell, 1992) was employed in choosing a smaller contingent of five course participants for pre- and post-course interviews each year for the collective case-study portion of this research. Criteria for selection included the participants' experiences in working with ELLs, their own language backgrounds, and most importantly, whether or not they would be involved in practicum or pre-practicum experiences in the spring semester of the school year – information they provide on the pre-survey that was administered on the first day of the methods class. Their selection was a diverse grouping of those persons with extensive ELL experiences as well as others who had little experience working with ELLs or learning a second language. One of the 2009 preservice teachers who

began this portion of the study, later withdrew. Attention was given to matching the demographics of the 2010 participants as closely as possible to the 2009 sub-group for optimal comparison. The data from the 2010 sub-group was chosen for the detailed case-study analyses featured in Chapters Five and Six of this dissertation.

Data Collection Procedures

Data collection for this study was conducted during the fall 2009 and fall 2010 semesters at the university. Quantitative data was collected through the pre- and post-surveys, and qualitative data was gathered through “interviewing, observing and reviewing material culture” (Rossman & Rallis, 2003, p. 139) – typical for case studies. Procedures for this collection are noted in Tables 3.1 and 3.2.

Table 3.1. Course-Participant Data Collection & Analysis Procedures

Data Sources	Description	Data Analysis
Pre-surveys	<ul style="list-style-type: none"> • Pre-survey, fall 2009 (n = 16) • Pre-survey, fall 2010 (n = 13) 	<ul style="list-style-type: none"> • Descriptive statistics • Paired t-tests • Correlations • Multiple regression analyses
Post-surveys	<ul style="list-style-type: none"> • Post-survey, fall 2009 (n = 15) • Post-survey, fall 2010 (n = 13) 	<ul style="list-style-type: none"> • Descriptive statistics • Paired t-tests • Correlations • Multiple regression analyses • Multiple coding methods for open-response questions focused on participant inquiries for further training and detailed descriptions of strategies/concepts learned

Table 3.2. Multiple-Case Study Data Collection & Analysis Procedures (n = 5)

Data Sources	Description	Data Analysis
Pre-interviews	<ul style="list-style-type: none"> • Conducted at the beginning of the semester 	<ul style="list-style-type: none"> • Multiple coding methods focused on participants' knowledge of academic-language strategies and experience with ELLs

Table 3.2. (*continued*)

Data Sources	Description	Data Analysis
Course Assignments	<ul style="list-style-type: none"> • Critical Analysis • Case Analysis • Lesson Organizer/Reflection • Concept Presentation 	<ul style="list-style-type: none"> • Multiple coding methods focused on the discussion of strategies that benefit ELLs
Post-interviews	<ul style="list-style-type: none"> • Conducted at the end of the semester or thereafter 	<ul style="list-style-type: none"> • Multiple coding methods focused on participants' internalization of academic-language strategies from course
Academic Language Projects	<ul style="list-style-type: none"> • Projects submitted to the Practicum Office, designed to assess students' capability to design instruction for ELLs (2009: n = 13; 2010: n = 11) 	<ul style="list-style-type: none"> • Multiple coding methods focused on the discussion of strategies that benefit ELLs

Instrumentation

Surveys

Pre- and post-surveys using Likert-scale items were administered to all participants enrolled in the secondary mathematics methods course for that term (see Appendices A, B, C, and D). These questions inquired about the participants' demographic information, teaching experience, and language experience, while the post-survey featured open-ended questions that asked participants to offer more detailed reflections about their experiences in the course as it pertained to the academic-language intervention. In addition, the surveys included drawings regarding the preservice teachers' own experiences as mathematics learners on the pre-surveys and regarding their desired classroom structure on the post-surveys. Drawings were added to the surveys as Haney and his colleagues (2004) found, "Student drawings provide a rich opportunity to document students' perspectives and to transcend assumption and artifice regarding what is going on in classrooms" (p. 267).

The creation of the 2009 pre-survey (Appendix A) involved the melding of several different resources. For the purpose of continuity within the larger content-and-language

integration study, the researcher began with a pre-survey that was adapted from a study by O'Brien (2009) for the history arm, and subsequently redirected the content-specific references towards mathematics. Questions regarding participants' views about mathematics were gleaned and adapted from prior research conducted by the methods instructor, as their validity has already been tested and proven. The drawing portion of the pre- and post-surveys were adapted again from previous work of the methods instructor and also Mewborn and Cross (2007). The researcher developed the open-ended questions in the post-survey. Spangler (1992) notes, "As students ponder their responses to [open-ended] questions, some of their beliefs... will be revealed" (p. 19); thus, as the need arose to learn more details about the participants' experience with the interventions in their own words, open-ended questions became a necessary addition to the post-survey.

Interview Protocols

All preservice teachers selected for interviews will participate in semi-structured pre- and post-meetings during the fall semester of their enrollment in the methods course. As recommended by Lather (1986), these interviews were conducted in "an interactive, dialogic manner" (p. 266), wherein "researcher and participant together develop a more complex understanding of the topic" (Rossman & Rallis, 2003, p. 182). The pre-interview protocol (Appendix E) was modified from the history-and-language version of the larger content-and-language integration study and focused upon the demographic information, language experiences, and content philosophies of the interview participants, as well as their initial thoughts about teaching ELLs. The pre-interview meetings were also used to inquire about discrepancies on participants' pre-surveys and to request elaborations for various responses on the instrument. The interactions were semi-structured because the

necessity for additional questioning that may emerge based upon the participants' responses.

The post-interview protocol (Appendix F) was developed by the researcher. The goal of these questions was to obtain details about the interview participants' experiences with the language intervention in the course. Transcripts from their pre-interviews and course artifacts were analyzed for common themes and major discrepancies in order to develop the post-interview protocol. The post-interviews remained semi-structured and dialogic, as the researcher was present enough in the classes to move "from the status of stranger to friend and thus able to gather personal knowledge from subjects more easily" (Lather, 1986, p. 263).

Observations of methods course

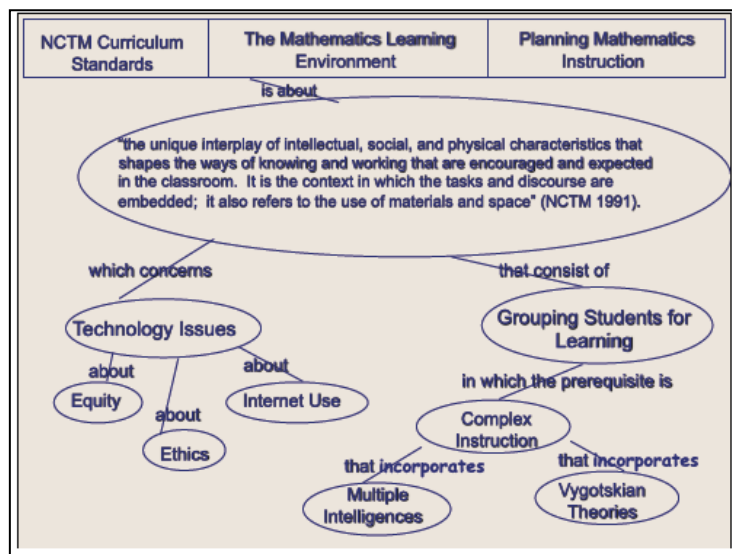
The researcher observed and audio-recorded the course sessions in which academic-language pedagogy was to be explicitly discussed or in which an "Imagine If" scenario is to be administered, resulting in seven recordings for fall 2009 and nine for fall 2010. The goal of this portion of the study was to document the academic-language strategies and pedagogy that the participants were taught through the language intervention and to "gather information where the group works and lives" (Creswell, 2007, p. 71) – in the methods course itself. As Rossman and Rallis (2003) purport, "The challenge [in conducting observations] is to identify the 'big picture' while noting huge amounts of detail in multiple and complex actions" (p. 195).

In observing the secondary mathematics methods course, the researcher collected handouts distributed to the class for each class session the attended. The handouts included copies of the lecture-note slides, extra copies of any activities the preservice

teachers can use in their own practices, and occasionally, additional articles. Digital audio recordings were made for these sessions and transcribed as needed. Finally, field notes were logged on computer for each class in order to keep a running record of the details of the classes' proceedings and to document the observer's "emotional reactions to events, analytic insights, questions about meaning, and thoughts for modifying [the study's] design" (p. 196). Seven classes were observed in the fall of 2009, and nine were observed in 2010. It should be noted that in the 2010 semester, field notes were not logged for one complete class session and parts of two others as the researcher assisted in the instruction of the classes during those times.

The methods course instructor utilized a class routine of giving an introduction to the class, situating the day's topic with the aid of a lesson organizer (see Figure 3.2), then proceeding to into a mini-lecture which is theoretically and practically based.

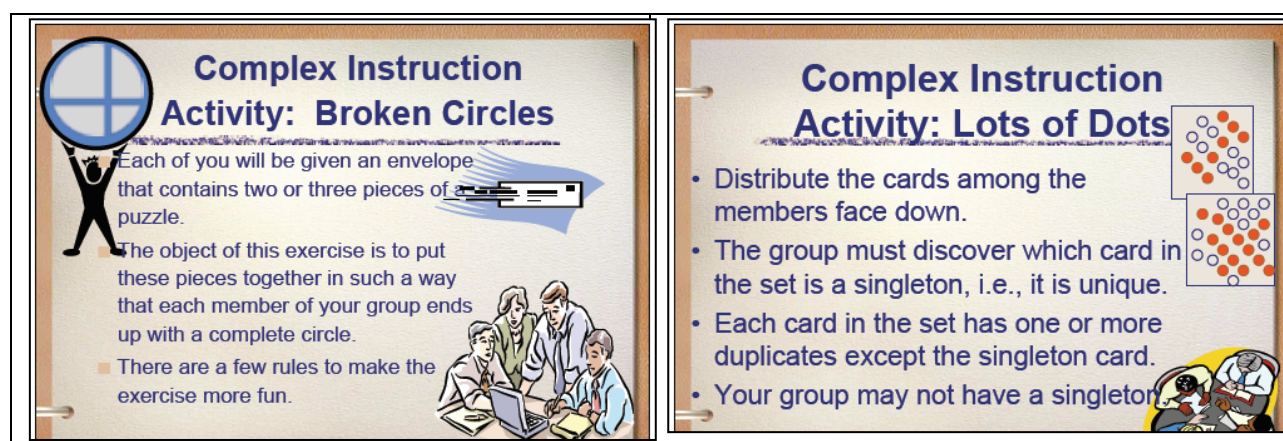
Figure 3.2. *Lesson Organizer, 9/27/10 Lecture*



Most mini-lectures were about 15-20 minutes, with some variation as the instructor entertains questions from the course participants as they emerge. Following the lecture,

the instructor facilitated activities designed to provide the preservice teachers with hands-on experiences with the concepts being discussed and with tools to use within the classes that they go on to teach themselves (see Figures 3.3).

Figure 3.3. *Sample In-class Activities, 9/27/10 Lecture*



A discussion of the task and of its implications for future instruction followed each activity. Depending on the time remaining in the course, the instructor may have begun a second mini-lesson and subsequent activity. If there was an assignment due at the next class session, the instructor allotted about ten minutes at the end of the current class to discuss the details of the assignment and answer any questions the preservice teachers may have.

Artifacts

The primary artifacts, or *material culture* (Rossman & Rallis, 2007), involved in this study are each participant's "Imagine If" activities, the Case Analysis assignment, the Lesson Organizer/Reflection assignment, the Concept Presentation assignment, and the Academic Language Projects (ALPs). The "Imagine If" activities were developed by the methods-course professor in order to periodically assess the preservice teachers' thinking about teaching ELLs (see Figure 3.4).

Figure 3.4. Sample “Imagine If” scenario, 11/1/10 Lecture

Mathematical Task Questions, Imagine If...

Grab a sheet of paper. How many layers of paper are there? Fold the paper in half. How many layers of paper are there? Record your findings on the table. Continue folding; then, fill out the table, graph the table's data, write a function rule, draw a picture, identify the domain/range and independent/dependent variables, and describe the rate of change.

What are some of the ways students might solve this problem? What misconceptions might they bring? What might a teacher need to do to prepare to use this task with students?

- If the lesson involves whole class discussion, how will you ensure ELLs and other diverse learners participate in the discussion?
- This activity requires a written component? How will you help ELLs and other diverse learners complete the activity?

Four were given and collected each fall semester for each participant in the study (2009: 4 × 16 students; 2010: 4 × 13 students). The teacher educator maintained the same themes within the activities each year for the purpose of continuity in the research.

Seven assignments were given in the fall of 2009 and again in 2010. The Case Analysis and Lesson Organizer/Reflection assignments were collected for all participants as these two assignments contain specific prompts regarding the participants' beliefs and known pedagogical practices that address academic language and the instruction of ELLs. The Concept Presentations for all groups were observed in order to assess the preservice teachers' acquisition of academic-language instructional strategies from the course. Other assignments were gathered only as the preservice teachers offer comments, concepts, or strategies related to teaching ELLs.

Finally, in association with another arm of the larger content-and-language integration study, the Practicum Office at the university required the submission of an ALP for each pre-practicum semester. The ALP required preservice teachers to either observe classes and write journal entries regarding the language demands of the lessons, or to generate language objectives for their of their own class sessions. A total of ten of these

writing pieces must be assembled and submitted along with a log of the ten activities and reflection page. These data were collected for all participants in each term (16 in 2009, 13 in 2010).

Data Analysis

Anfara, Brown, and Mangione (2002) claimed, “The purpose of analysis is to bring meaning, structure, and order to data” (p. 31). Drawing on the “mixed” data sources (Greene & Caracelli, 2003) gathered in throughout the study, a constant comparative method (Charmaz, 2000) was employed to uncover from the data and verify what participants learned about teaching mathematics and the academic language of this discipline to ELLs. The overall procedures for analyzing the data collected for this study were outlined in Table 3.1. The examination of the quantitative data was performed using SPSS, a software program for advanced statistical analysis. Descriptive statistics were employed to identify overall item response percentages and possible trends or discrepancies. Correlations were utilized to examine the possible relationship between the preservice teachers’ views of their preparedness to teach ELLs, their attitudes about teaching ELLs in mainstream classes, their knowledge of effective methods for teaching ELLs, their views of the role of the mathematics teacher, and their perception of the nature of mathematics. Paired t-tests were performed to measure any differentiations between the pre- and post-surveys’ Likert-scale items. Regression analyses were conducted to determine which variables might influence the participants’ beliefs about teaching ELLs in mathematics.

Comparative analysis was applied to the syllabi from the 2008, 2009, and 2010 fall semesters to determine the resulting alterations of the methods course due to the language

intervention, as outlined by the methods instructor. The assessment of the qualitative data utilized Hyper RESEARCH, a coding and data management software package, as the primary data analysis tool. The interview transcriptions, methods-course observations and field notes, course assignments, “Imagine If” activities, and ALP submissions were coded using the software program and sorted according to their relevance to the conceptual framework. Constant comparative analysis allotted for the emergence of thematic categories across the codes and within each element of the framework in order to understand and generate inferences from the data (Anfara et al., 2002). Much of the data was tabulated in order to “enhance the opportunity for criticism and public inspection of [the study] – to encourage analytic openness” (p. 33), as well as increase the validity of the research. The survey drawings were examined using a holistic review, wherein patterns were searched for and assessed from the pre-semester survey to the post-semester survey and also across participants (Haney et al., 2004). The analysis process was iterative, and triangulation was maintained through the consistency of codes employed across the various sources of data.

Design Limitations

There were a number of limitations to the design of this study. The first was the question of the generalizability of the study, as it involved one method of instruction for preservice teachers for one course that is offered once during the school year and this study was conducted at only one university site. The course’s number of enrollees was less than 20 people in both 2009 and 2010, which is the typical population of the course at the university, but neither adequately represents the current population of preservice teachers nor practicing secondary mathematics teachers. Another limitation was the withdrawal of

case-study participation. In 2009 one preservice teacher withdraw from the case study and only two Academic Language Projects were able to be obtained, which, in light of the focus of this dissertation on the 2010 cohort, did not affect the data presented in Chapters Four, Five, and Six. However, in 2010 one preservice teacher decided not to continue with the study and another decided to change majors altogether.

In addition, three of the participants did not submit Academic Language Projects, which proved to be plentiful resource for some the coding citations and thus may have altered the results of the study. A third restraint was that the 2010 case-study participants were interviewed and their artifacts coded and analyzed, but due to the timeframe of a dissertation, as well as the time involved in scheduling meetings with 13-16 people as well as coding and examining all their work in detail, this was not a doable task. Finally, the researcher was not able to observe pupil impact of the lessons submitted by the case-study participants; a necessary next step in this line of research.

Chapter 4

QUANTITATIVE RESULTS & ANALYSIS:

PARTICIPANTS' PERCEPTIONS AND PREPAREDNESS TO TEACH ELLS

As outlined in the previous chapter, pre-semester and post-semester surveys were administered to both the fall 2009 and fall 2010 cohorts in order to gather the preservice teachers opinions about teaching ELLs and to measure any change in their views that could possibly be attributed to the content-and-language intervention integrated into the course. This chapter relays the survey results and analyzes their significance. In particular, these results address the previously stated research question and the second sub-question:

How does a content-and-language integration intervention effect the preparation of secondary mathematics preservice teachers in order to improve instruction for diverse learners, more specifically ELLs, in mainstream mathematics classes?

- a. Do the preservice teachers' beliefs change regarding their ability to provide content and language-integrated learning opportunities for ELLs in mathematics? If so, how?

Both the questions of the survey and the processes involved in the analysis of the survey data are explicated. Descriptions of the pertinent variables are provided, and the handling of indecisive survey responses is detailed. In addition, descriptive statistics, paired t-tests, and regression models are conducted, and conclusions are offered from these calculations. The chapter ends with an interpretive summary of the survey results in

order to situate these results within the research regarding the preparation of preservice teachers to address the mathematics register at the secondary level.

Survey Results

Descriptive Statistics

In order to obtain a preliminary synopsis of the participants' views and to determine possible patterns or abnormal occurrences, descriptive statistics were compiled for each survey question using Microsoft Excel and later SPSS 18.0. Central tendencies (mean, median, and mode) were calculated in Excel for the pre-semester and post-semester surveys, both altogether and individually. These results were confirmed in SPSS, wherein the standard deviation was added to the statistics, as well as the normalization curve. The generated data were compared to the desired results, based on a four-point Likert scale, for each question in order to identify discrepancies and determine survey items that required further study. For instance, the desired response for the first survey question, "I am comfortable with planning mathematics-content objectives for my classes," was "agree" (3) or "strongly agree" (4). The pre-survey calculation for the mean, median, and mode were 3.04, 3, and 3, respectively – all in the "agree" range of responses – and the post-survey statistics were 3.43, 3, and 3, respectively. Because all of the results were within the span of the desired responses, Question 1 was not noted for further study.

Through this examination, it was discovered that there were a number of questions whose data began in the undesirable range, then shifted to the desirable interval by the end of the semester. For example, participants' responses on Question 3 of the pre-survey ("I feel confident in my ability to instruct all students, especially ELL students, in how to interpret the language of mathematics and how to communicate within this language

effectively”) averaged to 2.59 (mean), 3.00 (median), and 3 (mode). The mean was initially lower than the desired response of 3 or 4; however, by the post-survey, the mean changed to 3.07. Any survey items whose statistics did not correspond to the preferred answers were noted for further study. Table 4.1 lists the questions that were selected for continued examination due to this criterion. It should be noted that Question 15 (“I believe that ELL students should be included in mainstream mathematics classes only when they have attained native-like fluency in English.”) and Question 28 (“In mathematics, problems can be solved without using a particular algorithm or rule.”) also showed marked differences from the desired responses. However, these questions were slightly altered between the 2009 and 2010 versions of the survey, and it was found through descriptive statistics and paired t-tests that the structural changes were significantly different. Therefore these two questions were removed from the survey items of interest. In addition, Questions 29, 30, and 31 from the 2010 surveys show some differences in their means from the desired responses. Due to the low number of participants ($n = 13$) and resulting low level of statistical power, these discrepancies were not included in the list of survey questions to continue investigating.

Table 4.1. *Questions of Interest Based on Descriptive Statistics*

Survey Item	Mean (pre, post)	Median (pre, post)	Mode (pre, post)	Desired response
Q12. I have adequate training to work with ELL students in a secondary mathematics classroom.	2.11, 2.92	2, 3	2, 3	3-4
Q16. It is my belief that ELL students should be able to use their native languages in any class.	2.87, 2.96	3, 3	3, 3	3-4

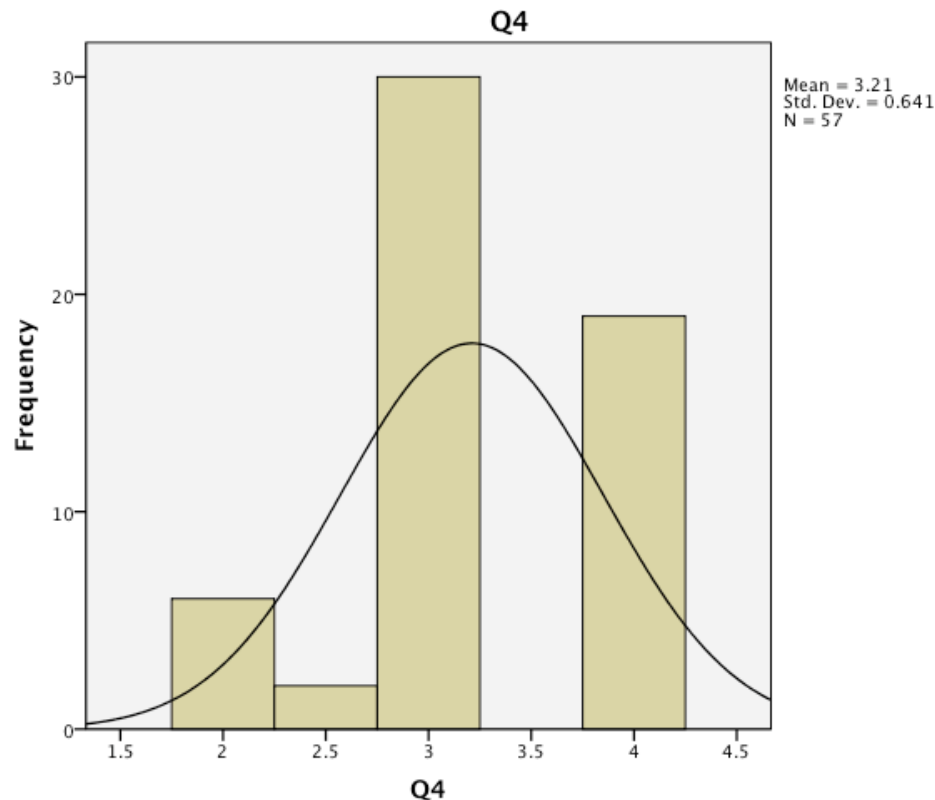
Note. 1 = strongly disagree, 2 = disagree, 3 = agree, 4 = strongly agree

Undecided Responses

As noted in Table 4.1, the participants were given four choices for each question – “strongly disagree,” “disagree,” “agree,” and “strongly agree.” A “neutral” option was not included in order to encourage the preservice teachers to adjudicate each topic rather than resorting to an indecisive choice. However, the participants still chose to submit answers such as “2/3” or “2 or 3.”

Initially, indecisive responses were replaced with “2.5.” Computationally, this did not seem to have much effect, with the exception of the standard deviation. However, when the histograms and normalization curves were generated in SPSS, major disturbances were noticed in the upper portion of the curve. (See Figure 4.1 for a sample display.) In addition, some of the undecided responses were converted to abnormal data when imported into SPSS from Excel. For example, “2/3” caused the data for a question to be classified as a string rather than numeric. When the data was converted to numeric, “2/3” became “13516070400” or “40570.” In one case, there was a response of “1/2” which translated to “13513305600” in SPSS. There were so few of these kinds of responses that it was deemed appropriate to report them as “missing” responses and ignore them in the statistical analysis.

Figure 4.1. Histogram and Normalization Curve for Question #4 using “2.5”



Paired t-tests

Paired t-tests were conducted on the survey items in order to determine the differences in item means between the pre- and post-semester questionnaires that were statistically significant ($p < .05$). As mentioned in previous chapters, survey items embodied five themes: preservice secondary teachers' perceptions of their own *preparedness* to teach ELLs; their *knowledge* about modifications and accommodations for ELLs; their *attitudes* about teaching ELLs and opinions about these pupils' inclusion in mainstream classes; the *role* of the mathematics instructor in the language-acquisition process; and their *philosophies* regarding the teaching, learning, usefulness, and nature of mathematics. The t-tests were run in thematic groups in order to gain a glimpse into which

categories may have been impacted more extensively by the content-and-language intervention.

From the *preparedness* group, the queries that produced significant differences from the beginning of the semesters to the end were Questions 1, 3, 12, 21, and 23. In the *knowledge* grouping, Question 9, 17, and 18 produced significant differences. Question 15 in the *attitudes* category showed significant differences in the means; however, this was a question that was altered between the two years and thus, this question was not investigated any further. Within the *philosophy of mathematics* category, Question 14 was the only query that generated a difference of significance. Finally, from the *mathematics teacher's role* group of questions, all four questions – Questions 10, 13, 22, and 24 – showed significant changes in their means. It should be noted that Question 30 from the 2010 surveys showed statistically significant changes, but again, due to its lack of power, the item was not investigated further.

To narrow the survey items to be examined further, the paired t-tests were performed again and split according to years. Questions that showed statistical significance in both years were selected for more detailed study. Table 4.2 catalogs these questions according to their theme, mean from the pre-semester survey to the post-survey, and overall t-test results.

Table 4.2. *Questions of Interest Based on Paired t-tests*

Survey Item	Theme	Mean (pre, post)	Overall t-test Results ($p < .05$)
Q12. I have adequate training to work with ELL students in a secondary mathematics classroom.	Preparedness	2.12, 2.79	$t = -4.612$, $p = .000$
Q14. Mathematics is a fluid subject wherein you can be creative and discover things for yourself.	Mathematics philosophy	3.12, 3.50	$t = -3.434$, $p = .001$

Table 4.2. (*continued*)

Survey Item	Theme	Mean (pre, post)	Overall t-test Results ($p < .05$)
Q17. Teachers should not modify their instruction for the ELL students enrolled in mainstream mathematics classes.	Modification knowledge	2.04, 1.64	$t = 3.667$, $p = .001$
Q22. Language development should only be taught within English courses.	Mathematics teacher's role	1.81, 1.37	$t = 3.667$, $p = .001$
Q23. As a mathematics teacher, I intend to concentrate exclusively on mathematics-content objectives for my classes.	Preparedness	1.93, 1.71	$t = 1.800$, $p = .042$

Variables

For the remainder of this chapter, the survey items that were designated for further investigation are referred to as variables and are renamed according to the attributes being examined. This will allow for discussion of the results without listing entire questions as in Tables 4.1 and 4.2. Descriptions of the concepts denoted in the surveys and the variables assigned to them follow and are outlined according to five themes analyzed by the surveys.

Preparedness. The preservice teachers' perceptions of their own preparedness to teach ELLs were captured in Questions 12 and 23. These two variables, *adequate ELL training* and *objectives concentration*, analyze the participants' feelings about their preparation to address the needs of ELLs in their instruction and to develop language objectives in addition to their mathematics-content objectives for this instruction.

Mathematics Philosophy. Question 14, *mathematics as fluid*, speaks to the preservice teachers' philosophies regarding the nature of mathematics. Other questions within this theme addressed how mathematics is learned and should be taught. However, these survey items did not show any pronounced changes or discrepancies.

Attitudes about ELLs. Participants' attitudes about teaching ELLs and their opinions about these pupils' inclusion in mainstream classes were measured in the questions related to this theme. Question 16, *native language usage*, gauged the preservice teachers' thoughts about the appropriateness of ELLs' use of their native languages within mainstream classes. The responses were lower than the minimum desired response of 3 ("agree"); thus, this variable will require careful examination.

Modification Knowledge. The survey items for this theme addressed the teachers' knowledge and opinions about modifications and accommodations for ELLs. The appropriateness of *instructional modification* was the focus of Question 17. Other related items inquired about additional time allotment for ELLs in mainstream classes, but no distinctions were observed in the responses.

Role of Mathematics Teacher. The role of the mathematics instructor in the language-acquisition process was addressed in the last grouping of questions. Specifically, Question 22, *isolated language*, queried the inclusion of language development in other courses, such as mathematics, outside of an English course. This was an important indicator as to how the preservice teachers might value the content-and-language intervention; therefore, requiring thorough analysis.

Correlations

In order to determine possible relationships between pairs of the variables noted in the previous section, bivariate one-tailed Pearson's correlations were performed at the .05 alpha level. Significant correlations were detected between five pairs of the variables as noted in Table 4.3. Overall, the findings indicate that there is a relationship between whether or not one of the preservice teachers from the methods course intends to employ

more than content objectives in their classrooms and his or her feelings about receiving adequate training to instruct ELLs, about whether or not native-language use is appropriate in mainstream mathematics courses, about the modification of instruction for the benefit of ELLs, and about language development's inclusion in content areas besides English. As the number of entries in this statistical test was small, fluctuating between $n = 52$ and $n = 57$, Kendall's tau and Spearman's rho were also conducted and generated comparable results.

Table 4.3. *Overall Relationships among Selected Variables*

Survey items	1	2	3	4	5	6
1. Adequate ELL training		.119	.042	-.128	-.263*	-.278*
2. Mathematics as fluid			.046	-.129	-.137	.005
3. Native language usage				.073	-.171	-.251*
4. Instructional modification					.034	.248*
5. Isolated language						.331**
6. Objectives concentration						

* Correlation is significant at the 0.05 level (1-tailed).

** Correlation is significant at the 0.01 level (1-tailed).

The strongest correlation is that between *isolated language* and *objectives concentration* ($r = .331$, $p < .01$), indicating that the more committed a participant was to including language development in his or her instruction, the more likely he or she would also utilize language objectives in addition to content objectives. The correlation tests indicate that *instructional modification* and *objectives concentration* ($r = .248$, $p < .05$) are related in a similar manner, although with slightly less strength, in that as a preservice instructor noted a belief in making instructional modifications for ELLs, he or she was also likely to believe in the inclusion of both content and language objectives in instructional

planning. The pairings of *native language usage* and *objectives concentration* ($r = -.251, p < .05$) and of *adequate ELL training* and *objectives concentration* ($r = -.278, p < .05$) corresponded in parallel manners, as the more negative participants felt about focusing solely on content objectives, the more positive they deemed the usage of native language in mainstream class and their preparation to teach ELLs in their future classrooms. The only other significant correlation that emerged in the analysis was between *adequate ELL training* and *isolated language*. The data, $r = -.263$ and $p < .05$, suggests that as the preservice teachers felt more confident about their ability to instruct ELLs, they were less likely to agree with relegating language development to English courses.

Split-level correlation tests were executed per pre- and post-semester survey, and again per cohort year. In the pre-survey results (see Table 4.4), the *isolated language/objectives concentration* relationship was confirmed ($r = .365, p < .05$). However, an additional correlation emerged between *adequate ELL training* and *mathematics as fluid* ($r = -.338, p < .05$), implying that as participants' confidence levels regarding teaching ELLs increased, their belief in the fluid nature of mathematics declined. It is far more probable that there is no relationship between these two variables, confirmed by the lack of significant correlation between them in the post-survey results ($r = .114, p = .290$). Also, corroborated in the post-survey analysis (see Table 4.5) was the relationship between the *instructional modification* and *objectives concentration* variables ($r = .362, p < .05$). With larger numbers of participants, split-level analyses would have been conducted for each year's survey results. However, due to the lack of power for the existing sample, this level of examination was excluded from this study.

Table 4.4. *Relationships among Selected Variables, Pre-Semester Surveys*

Survey items	1	2	3	4	5	6
1. Adequate ELL training		-.338*	-.028	-.009	-.148	-.249
2. Mathematics as fluid			.109	-.014	-.099	.180
3. Native language usage				.112	.104	-.302
4. Instructional modification					-.223	.097
5. Isolated language						.365*
6. Objectives concentration						

* Correlation is significant at the 0.05 level (1-tailed).

Table 4.5. *Relationships among Selected Variables, Post-Semester Surveys*

Survey items	1	2	3	4	5	6
1. Adequate ELL training		.114	-.005	.144	-.027	-.221
2. Mathematics as fluid			-.041	.000	.149	.000
3. Native language usage				.142	-.314	-.210
4. Instructional modification					.089	.362*
5. Isolated language						.214
6. Objectives concentration						

* Correlation is significant at the 0.05 level (1-tailed).

Regression Analysis

This section presents the results of conducting ordinary least squares multiple regression tests for the variables selected for the correlation tests using SPSS 18.0.

Objectives concentration was entered as the outcome variable, as the overall correlation tests indicated significant relationships between this variable and four of the remaining five variables. In order to produce the outcome variable, the means of the other five variables – *adequate ELL training*, *mathematics as fluid*, *native language usage*, *instructional*

modification, and *isolated language* – were utilized as predictor variables. Although, there was no significant correlation produced between *objectives concentration* and *mathematics as fluid*, the latter was included in the regression analysis in order to confirm the lack of relationship between the two variables.

The outcome variables were selected for regression in order of their correlation strength and in descending order. Thus, *isolated language* was entered first, followed by *adequate ELL training*, *native language usage*, *instructional modification*, and *mathematics as fluid*, respectively. I theorized that the variation found in the preservice teachers' decisions to concentrate on language objectives in addition to content objectives could be accounted for by the aforementioned variables. In statistics terminology,

$$H_0 = \beta_{\text{isolatedlanguage}} = 0 ; H_0 = \beta_{\text{adequateELLtraining}} = 0 ; H_0 = \beta_{\text{nativelanguageusage}} = 0 ; \\ H_1 = \beta_{\text{isolatedlanguage}} \neq 0 ; H_1 = \beta_{\text{adequateELLtraining}} \neq 0 ; H_1 = \beta_{\text{nativelanguageusage}} \neq 0 ; \\ H_0 = \beta_{\text{instructionalmodification}} = 0 ; H_0 = \beta_{\text{matheamaticsasfluid}} = 0 ; \\ H_1 = \beta_{\text{instructionalmodification}} \neq 0 ; \text{ and } H_1 = \beta_{\text{matheamaticsasfluid}} \neq 0 .$$

The significance level was analyzed for one-tailed tests at the 0.05 level.

Single Predictors. The variance attributed to each predictor variable was computed and analyzed individually, as shown in Table 4.6. The statistics for the *isolated language* ($R^2 = .109$, $F = 6.632$, $p < .05$) indicate that this predictor variable accounts for about 10.9% of the variance in the *objectives concentration* variable, and *adequate ELL training* ($R^2 = .077$, $F = 4.256$, $p < .05$) accounts for about 7.7% of the outcome variable's variance. Both of these predictors' statistics were significant. In contrast, the remaining variables, *native language usage* ($R^2 = .063$, $F = 3.570$, $p > .05$), *instructional modification* ($R^2 = .062$, $F = 3.607$, $p > .05$), and *mathematics as fluid* ($R^2 = .000$, $F = .001$, $p > .05$) did not contribute significantly to the variance in *objectives concentration*. The correlation tests indicated that

there was not a significant relationship between the *objectives concentration* and *mathematics as fluid* variables; thus, the low attribution levels in the regression analysis between these two variables were expected. However, the lack of significance between *native language usage* and *instructional modification* was not expected, but, perhaps, can be explained by the low sample size ($n = 52-57$) and, thus, potential bias and low power.

Table 4.6. *Simple Regression Statistics (Objectives Concentration as Outcome Variable)*

Predictor Variable	R Square	Adjusted R Square	Unstdnd. Coefficient	Stand. Coefficient	F statistic	Sig. (F)
Isolated language	.109	.093	.353	.331	6.632	.013
Adequate ELL training	.077	.059	-.223	-.278	4.256	.044
Native language usage	.063	.045	-.223	-.251	3.570	.064
Instructional modification	.062	.044	.245	.129	3.607	.063
Mathematics as fluid	.000	-.019	.007	.005	.001	.970

Multiple Regression Model, Five Predictors. The regression of the outcome variable, *objectives concentration*, on the five predictor variables was significant [$R^2 = .257$, $F(5, 51) = 3.119$, $p < .05$]. Therefore, as the variance accounted for by the five variables together did vary significantly from 0, the null hypothesis must be rejected. The results of the model summary and significance levels are shown in Table 4.7, and the unstandardized coefficients are displayed in Table 4.8. *Isolated language* explained 10.1% of the variance, while *adequate ELL training* accounted for an additional 4.4%, *native language usage* an additional 4.1%, and *instructional modification* an additional 6.6%. However, the addition of the *mathematics as fluid* variable accounted for only an additional 0.5% of the variance. Altogether the predictor variables explain about 25.6% of the variance in the outcome

variable, leaving 74.4% of the variance inexplicable. In addition, the negative values of the coefficients for *adequate ELL training* and *native language use*, as reported in Table 4.8, were expected due to the opposite positions of the survey questions.

Table 4.7. *Model Summary and Significance of Five Predictors*

Predictor Variable(s)	R^2	ΔR^2	F	$p (F)$	DW
1. IL	.101	.101	5.494	.023	
2. IL, AET	.145	.044	4.068	.023	
3. IL, AET, NLU	.186	.041	3.587	.020	
4. IL, AET, NLU, IM	.252	.066	3.879	.008	
5. IL, AET, NLU, IM, MAF	.257	.005	3.119	.017	1.717

Note. Predictor variables: Isolated language (IL), adequate ELL training (AET), native language usage (NLU), instruction modification (IM), and mathematics as fluid (MAS); Durbin-Watson (DW).

Table 4.8. *Unstandardized Coefficients from Multiple Regression Model*

Predictor Variable	B
Isolated language	.262
Adequate ELL training	-.169
Native language usage	-.203
Instructional modification	.277
Mathematics as fluid	.095

According to the values in Table 4.8, the regression solution for this model was:

$$\hat{Y}_{objectivesconcentration} = 1.606 + 0.262X_{isolatedlanguage} - 0.169X_{adequateELLtraining} - 0.203X_{nativelanguageusage} + 0.277X_{instructionalmodification} + 0.095X_{mathematicsasfluid}.$$

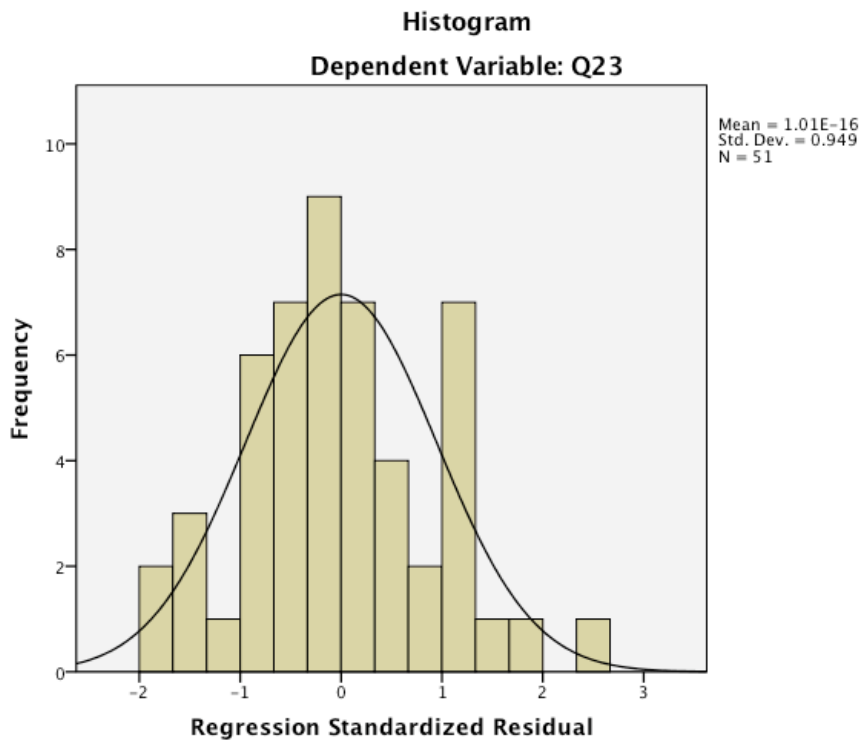
Thus, if the mean of each of the predictor variables was 0, the predicted mean of the *objectives concentration* variable would be 1.609. This result would imply that even without the influence of the predictor variables preservice teachers would not focus solely on

content objectives in planning their mathematics lessons. As the response choices ranged from 1 to 4, it would be impossible for *isolated language* to have a mean score of 0; consequently, the sum of the ratios of the mean scores for the predictor variables indicated in the regression equation could not be more than 0.301. Otherwise, the score of the outcome variable would increase beyond a mean of 2, which is undesired.

The Durbin-Watson statistic generally enables the determination of the dependence of one data point on the previous one (i.e. whether or not a lag-1 autocorrelation occurs). The DW statistic for this data set was 1.717, which does lie between the predicted minimum (1.21) and maximum (2.50) according to the residual statistics generated. This indicates that we must reject the null hypothesis and presume that, indeed, a lag-1 autocorrelation does exist. Tolerance, upon the addition of the fifth variable, was 0.963, and Variance Inflation Factor was 1.039; both statistics indicating minimum multicollinearity.

Multiple Regression Model, Four Predictors. While the F-statistic of the multiple regression model indicated significance in the inclusion of all five predictor variables, *mathematics as fluid* did not have a significant correlation to the outcome variable, as noted in Table 4.3, and the simple regression statistics in Table 4.6 also overwhelmingly indicated a lack of significance in the relationship between the two variables. Further, the histogram generated with the standardized residuals from all five potential predictor variables displayed a number of disparities in its normalization (see Figure 4.2). Therefore, the multiple regression tests were run again without the *mathematics as fluid* variable as a comparison.

Figure 4.2. Histogram and Normalization Curve for Outcome Variable Using Standardized Residuals for Five Predictor Variables



The regression of the outcome variable, *objectives concentration*, on the five predictor variables demonstrated a higher F-statistic and was more statistically significant [$R^2 = .252$, $F(4, 51) = 3.879$, $p < .01$]. Thus, as stated before, the null hypothesis must be rejected due to significant variance, accounted for by the four variables combined. The results of the model summary, significance levels, and unstandardized coefficients are shown in Tables 4.9 and 4.10. The percentages of the variance explained by the predictor variables did not alter, despite the absence of *mathematics as fluid*, and the predictor variables altogether accounted for about 25.2% of the variance in the outcome variable, leaving 74.8% of the variance unexplained. According to the new coefficient values, the regression solution for this model was:

$$\hat{Y}_{objectivesconcentration} = 1.932 + 0.254X_{isolatedlanguage} - 0.164X_{adequateELLtraining}$$

$-0.200X_{nativelanguageusage} + 0.267X_{instructionalmodification}$. In this scenario, the predicted mean of the *objectives concentration* variable would be 1.932 if the mean of each of the predictor variables was 0. Again, the implication is that even without the influence of the predictor variables, preservice teachers would not focus on language objectives in addition to content objectives in their lesson planning. However, in this four-variable scenario, the sum of the ratios of the mean scores for the predictor variables indicated in the regression equation could not be more than 0.068, else the score of the outcome variable would increase beyond the desired maximum mean of 2. Again, the negative values of the coefficients for *adequate ELL training* and *native language use* shown in Table 4.10 were expected due to the opposing natures of the survey questions.

Table 4.9. *Model Summary and Significance of Four Predictors*

Predictor Variable (s)	R^2	ΔR^2	F	$p (F)$	DW
1. IL	.101	.101	5.494	.023	
2. IL, AET	.145	.044	4.068	.023	
3. IL, AET, NLU	.186	.041	3.587	.020	
4. IL, AET, NLU, IM	.252	.066	3.879	.008	1.700

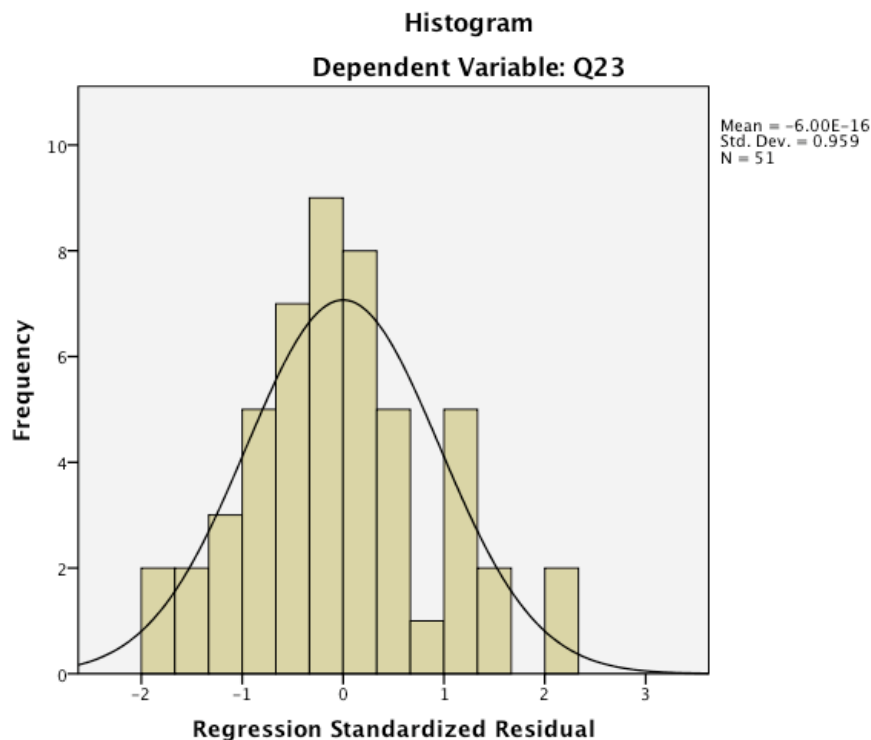
Note. Predictor variables: Isolated language (IL), adequate ELL training (AET), native language usage (NLU), and instruction modification (IM); Durbin-Watson (DW).

Table 4.10. *Unstandardized Coefficients from Multiple Regression Model*

Predictor Variable	B
Isolated language	.254
Adequate ELL training	-.169
Native language usage	-.200
Instructional modification	.267

The Durbin-Watson for the new model was 1.700, which does lie between the predicted minimum (1.16) and maximum (2.51) according to the residual statistics. This indicates that we must reject the null hypothesis and presume that, indeed, a lag-1 autocorrelation does exist. Tolerance, upon the addition of the fourth variable, was 0.989, and the Variance Inflation Factor was 1.012; both statistics indicating minimum multicollinearity. Examination of the standardized residual histogram (Figure 4.3) infers that this second regression model is more normalized and thus, it can be concluded that the four-variable regression model is stronger than when the *mathematics as fluid* variable is included.

Figure 4.3. *Histogram and Normalization Curve for Outcome Variable Using Standardized Residuals for Four Predictor Variables*



For the purposes of comparison, split-file regression analysis was conducted, using the stronger model of four predictor variables to produce the outcome variable. Table 4.11 highlights the unstandardized coefficients for each predictor, as well as the changes in the correlation, the F-statistic and its significance, and the Durbin-Watson statistic. In examining the pre- and post-semester regression analyses, the pre-semester survey predictors [$R^2 = .627$, $F(4, 25) = 3.247$, $p < .01$] were stronger than the results from the post-semester survey [$R^2 = .566$, $F(4, 26) = 2.474$, $p > .01$]. The F-statistic was greater and statistically significant at the 0.05 level for the pre-survey; however, the post-semester results were not significant. Also, for the pre-semester outcome variable, *isolated language* and *native language usage* were the major contributors, as they each accounted for 13.2% and 13.7% of the variance in the *objectives concentration*, respectively. In the post-survey, *instructional modification* was the only statistically significant contributor to the outcome variable, explaining 20.6% of the latter's variance. Both split-file categories produced Durbin-Watson statistics that lay between the predicted minima and maxima. This indicates that we must reject the null hypothesis for both cases and presume that each contains lag-1 autocorrelations.

Table 4.11. *Split-File Regression Results*

Predictor Variable	Pre-Semester		Post-Semester	
	<i>B</i>	ΔR^2	<i>B</i>	ΔR^2
Isolated language	.458*	.132	.062	.042
Adequate ELL training	-.244	.050	-.280	.047
Native language usage	-.536*	.137	-.195	.026
Instructional modification	.263	.076	.636*	.206

Table 4.11. (*continued*)

	<i>Pre-Semester</i>	<i>Post-Semester</i>
F statistic	3.247	2.474
Sig.	.033*	.076
DW	1.294	1.775

Note. Durbin-Watson (DW).

* Correlation is significant at the 0.05 level (1-tailed).

** Correlation is significant at the 0.01 level (1-tailed)

Post-Hoc Power Analysis. In order to determine and examine the power and effect size of the various regression models, univariate analysis of variance was conducted on each of the models. The first scenario with five predictor variables (see Tables 4.7 and 4.8) had an observed power of .601, which is moderate, and a very large effect size ($f_1 = \sqrt{\frac{\lambda_1}{np}} \approx 1.012$) for a sample size of 51. The second model with four predictors (Tables 4.9 and 4.10) produced a higher level of power, .846, which is desirable and a slighter larger effect size of approximately 1.058. Table 4.12 displays the power levels and corresponding effect sizes under each of the predictor scenarios for the split-file groups – pre-semester surveys versus post-semester surveys. For each case, the effect sizes are fairly large, due to the small samples sizes. The stronger power within the four-variable scenario was to be expected, based on the correlation and regression analyses in the previous sections.

Table 4.12. *Split-File Power and Effect Size Results*

Grouping	Sample size	<i>Five Predictor Variables</i>		<i>Four Predictor Variables</i>	
		Power	Effect size	Power	Effect size
Pre-semester survey	25	.233	.827	.307	.884
Post-semester survey	26	.278	1.110	.411	1.030

Interpretive Summary

Gebhard and her colleagues (2002) purport that teachers must be given the opportunity to consider their beliefs and assumptions regarding linguistically and culturally diverse students. The pre- and post-semester surveys employed in this study offered a mechanism for the preservice teachers enrolled in the secondary mathematics methods course such an opportunity. The purpose of this chapter was to report the results of these surveys in order to address one of the sub-questions for this study: Do the preservice teachers' beliefs change regarding their ability to provide content and language-integrated learning opportunities for ELLs in mathematics? The goal of using surveys as instruments was to capture changes in opinions of the participants in five thematic areas: preparedness, knowledge, attitudes, role of the teacher, and philosophy about mathematics. The analysis of the survey data began with descriptive statistics, and continued into paired t-tests, correlation tests, and simple and multiple regression analysis – all these to determine which variables demonstrated the most significant change and which may have influenced one another. This section presents an interpretive summary of these statistical analyses.

The descriptive statistics and paired t-tests offered an initial affirmative response to this inquiry, producing statistically significant changes in the desired direction for several of the variables. Of the nine questions designed to measure the participants' preparedness to provide content-and-language integrated learning opportunities, five showed notable improvement. Three of the four questions that addressed the preservice teachers' knowledge about teaching ELLs demonstrated significant progress. All of the inquiries about the participants' beliefs about their role as more than simply a content teacher, but

also a language teacher, indicated positive and marked change. One of eight questions about the nature of mathematics also showed a change to the preferred response; however, this may have been due to the influence of the course as a whole, rather than the intervention itself. The preservice teachers' response to the questions regarding their attitudes about teaching ELLs did not demonstrate any significant change, but remained in the desired range of the Likert scale. This may be due to their notion of social justice, which is a theme that is promoted in every education course offered at the college. Thus, this lack of change may be due to the participants' acceptance or internalization of the concept prior to entering the course, resulting in an already developed belief in equitable education opportunities for all students, including ELLs.

The regression analyses indicated the strongest relationship ($p < .01$) lying between the *isolated language* and *objectives concentration* variables, also found significant ($p < .05$) within the pre-semester survey subgroup. This finding indicated that the preservice teachers' beliefs in the responsibility for language instruction lying with all teachers, not just English teachers or ELL specialists, would lead to the consideration of more than content objectives for a lesson, but also language-related goals. The increase in significant with the inclusion of the post-semester's surveys indicated the influence of content-and-language integrated intervention in promoting the interconnectivity of these two variables. Interestingly, a statistically significant relationship was also found in the pre-survey portion of the data between the *adequate ELL training* variable and the variable pertaining to participants' views of mathematics as a fluid, ever-changing content area (*mathematics as fluid*). However, as how one feels about feeling prepared to teach ELLs has little to no connection to one's views about the nature of mathematics and as this was the only

indication of any connection in the correlation tests, no emphasis was placed on these results.

The purpose of the simple and multiple regression analyses was to further investigate the relationships among these six variables. The *mathematics as fluid* variable, again, did not have a strong relationship to the other variables and thus, caused discrepancies in the residual normalization (review Figure 4.2) in addition to having very little effect as a predictor variable ($p > .05$). Therefore, the dependent *objective concentration* variable was reexamined using only four variables – *isolated language*, *adequate ELL training*, *native language usage*, and *instructional modification* – although the latter two variables produced F-statistics that were slightly beyond the $\alpha = .05$ threshold in the simple regression models. The regression model produced was very strong ($p_F < .01$), with most of the variance attributed to the predictor variables being shouldered by *isolated language* (10.1%). For the subgroups, the pre-semester survey model was stronger than that of the post-survey, with the most significant contributions being from *isolated language* and *native language usage* in the former's model. These findings in totality seem to indicate the strongest relationships being among participants' beliefs in including more than just content objectives in their planning and instruction, in allowing pupils to use their native languages in class, and in integrating language development into mathematics courses. However, the post-hoc power and effect size analyses point to more research being needed to confirm this conclusion, as the desired power levels were produced only for the overall group and the effect sizes were very large.

Linguistically responsive pedagogy (LRP; Lucas et al., 2008) requires that preservice teachers learn to identify the language demands of their content area and classroom tasks.

This requires much more than teaching the more technical, Tier-three (Beck et al., 2002) terms of the subject matter. It requires also instructing students in the ways they are to communicate the content themselves. Such pedagogical techniques are what the intervention endeavored to develop in the methods course students, whether or not they chose to participate in the research. Lucas and Grinberg (2008) state, “teacher attitudes and beliefs about their students’ language uses and language learning have an impact on their expectations of ELLs, the nature of their interactions with ELLs, and their instructional practices” (p. 613). Thus, the surveys administered in the two cohorts served as ways to measure the effect of the intervention on the participants’ attitudes and beliefs regarding the challenge of teaching ELLs in mainstream mathematic classrooms. According to the analyses in this chapter, the intervention did, indeed, affect the preservice teachers’ views. The next chapter will examine artifacts from a five-person, multiple-case study for evidence of this change as well as whether or not the participants exhibit the skills necessary to implement their change in views.

Chapter 5

QUALITATIVE RESULTS & ANALYSIS:

PORTRAITS OF THE MULTIPLE-CASE STUDY PARTICIPANTS

Lucas and colleagues (2008) outline three domains of linguistically responsive pedagogy (LRP): learning about the ELLs in one's classroom, identifying the language demands of the class, and scaffolding learning for ELLs. Comprising the scaffolding domain, the heart of LRP, the authors purported seven pedagogical practices that, with the addition of Swain's (1985) promotion of observable measurements of students' language capabilities, can be summarized into three conceptual elements – comprehensible input (Krashen, 1981), student engagement (Lucas et al., 2008), and comprehensible output (Swain, 1985). These are the pedagogical ideals that the intervention sought to instill in the preservice teachers enrolled in the secondary mathematics methods course. The goal of this chapter, therefore, is to begin the examination of the artifacts gathered during the study in order to provide insight into the five participants' ideologies about teaching mathematics, in general, as well as teaching ELLs.

As outlined in Chapter Three, five students from each of the 2009 and 2010 semesters (numbering 16 and 13, respectively) were selected for pre-semester and post-semester interviews, formulating a multiple-case study within this research with the goal being for replication logic (Yin, 1994) to emerge. The logic in this study would be that of the content-and-language integrated intervention's influence in the preservice teachers' initial practices. Even more specifically, the five cases from the 2010 cohort will be explored in

this chapter, as the intervention that these participants experienced was enhanced from the 2009 pilot year to be more explicit in its instruction of LRP elements (Lucas et al., 2008).

This chapter consists of portraits, or “within-case” analyses (Creswell, 1998) of the five case-study students: Scott, Mabel, Bryce, Natalie, and Hank. These descriptions include biographical information generated from their surveys and interviews, as well as philosophical information regarding their pedagogical stances offered in their interviews and their Case Study Assignments. In addition, data from the survey questions more closely inspected in Chapter Four (see Table 4.2) will be included from the perspective of each participant in order to inform their overall descriptions and to highlight the relationship between the survey questions and other data in the study (Anfara et al., 2002). As the demographics and backgrounds of this sub-group of participants is far more diverse than the white, middle to upper-class, monolingual female profile that typically exists in the current teaching force (Beykont, 2002; Bunch, 2010), the interpretive summary of this chapter may offer intriguing insight into the variety of belief systems required to move education in the U.S. towards LRP.

“Effort and improvement are what should be assessed and rewarded...”

Scott was a graduate of one of the most prominent engineering schools in the U.S. and around the world. He double majored in electrical engineering and computer science, but upon graduation, enlisted in AmeriCorps and entered the field of education. He spent about eighteen months as an adult ESL instructor in a low-socioeconomic section of an urban city. In a neighboring area within the city, Scott taught mathematics for a general-education diploma program for adults. About two-thirds of his students were ELLs, and the

remaining pupils were from a local housing project. In addition, Scott spent his time before matriculating his graduate studies tutoring middle-school and high-school students, as well as coaching an upper elementary level (fifth and sixth grades) robotics team for an afterschool program.

Scott underwent some ESOL training through AmeriCorps and plans to obtain his certification for teaching ELLs while matriculating his graduate studies (although he had not taken any courses for such prior to the fall 2010 semester). This may be the reason that he classified himself as being adequately prepared to teach ELLs on both his pre- and post-semester surveys, as noted in Table 5.1. He reported having some fluency in Japanese, having studied the language throughout high school. He did a “home stay” in Kyoto while in high school, and also worked for a software company in Tokyo the summer after he graduated from college. Scott describes his fluency in Japanese according to the mode of communication:

“Speaking I’d say I’m pretty good. I can hold a conversation. I’m almost illiterate because of the Japanese language uses Chinese characters, and there’s two thousand of them. So, I used to know about two hundred; now I, the ones I remember are about fifty. And then I can, I can write Japanese using their, sort of, their elementary alphabet that they, that they teach their kids in, like, first grade, before they learn the Chinese alphabet that all the adults use. So I can, I could write Japanese but I would look like I was a six-year-old.”

Perhaps, Scott’s fluency would appear a bit older than six years old, as “a Japanese kid cannot read a newspaper until they turn 11 or 12 because of all the Chinese characters” either.

Table 5.1. *Scott's Responses to Questions of Interest*

Survey Item	Theme	Mean (pre, post)
Q12. I have adequate training to work with ELL students in a secondary mathematics classroom.	Preparedness	3, 3
Q14. Mathematics is a fluid subject wherein you can be creative and discover things for yourself.	Mathematics philosophy	3, 4
Q17. Teachers should not modify their instruction for the ELL students enrolled in mainstream mathematics classes.	Modification knowledge	1, 1
Q22. Language development should only be taught within English courses.	Mathematics teacher's role	1, 2
Q23. As a mathematics teacher, I intend to concentrate exclusively on mathematics-content objectives for my classes.	Preparedness	1, 2

Scott's pedagogical philosophy, written in the earlier part of the fall semester, stated, "the core of successful mathematical pedagogy is comprised of two pillars: high expectations for effort and behavior, and teaching methods that involve the students and present mathematics in real-world context." While he mentioned the use of modifications for ELLs as an aspect of "good teaching," supported by his responses in both the pre- and post-surveys to Question 17 (see Table 5.1), he also emphatically pointed to *Stand and Deliver's* acclaimed Calculus teacher, Jaime Escalante, as a model for maintaining high expectations of all students.

"No free rides, no excuses ... is a good philosophy in general. If teachers lower their expectations of urban students because they see them as victims of their circumstances, they are actually doing their students a great disservice."

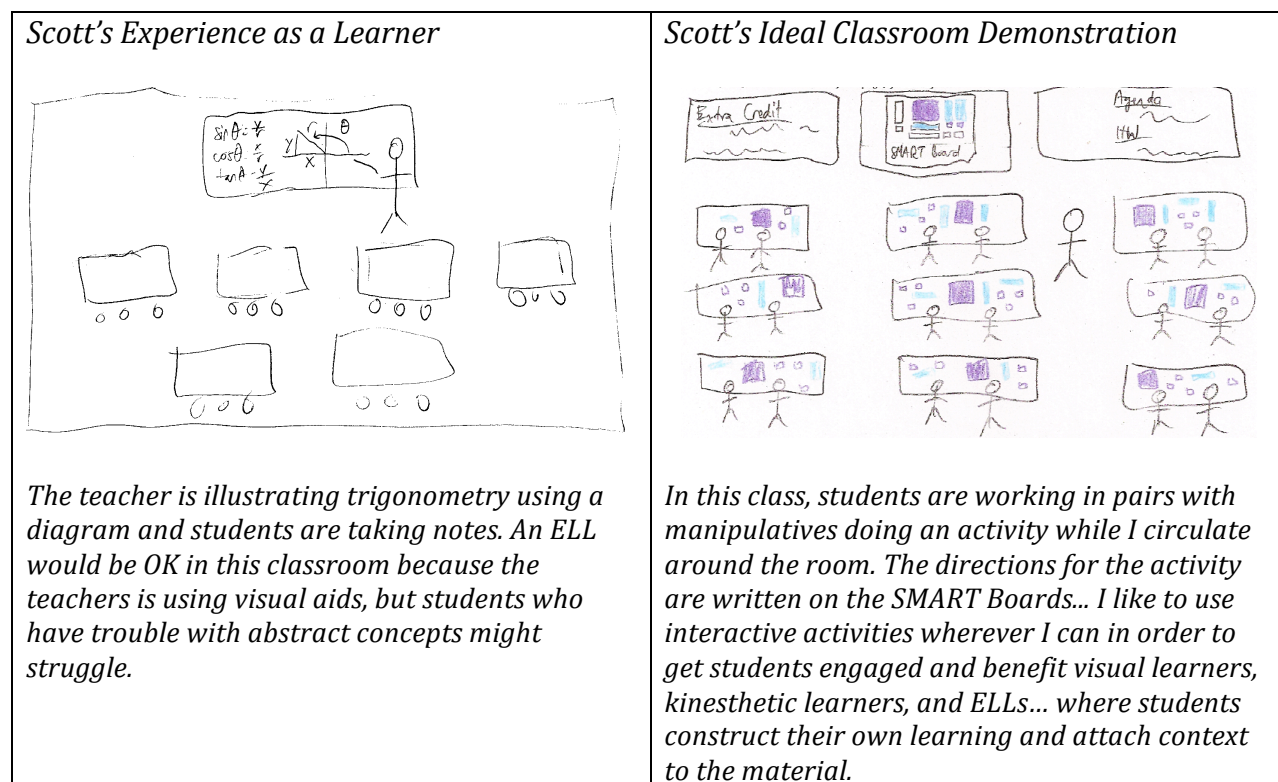
Scott argued for making content relevant for students in order to involve them in their own learning, adopting Vygotsky's notion of the internalization of knowledge by the attachment

of meaning to learning. He believed that this can be accomplished through the continuous linking of mathematics concepts to real-world applications.

“Students who sit through math class taking notes, regurgitating facts, and learning recipe-type procedures without any context will almost inevitably forget everything they have ‘learned.’ Math concepts should always be presented with a real-world application, so that students can see that math can actually be useful and not merely something they are forced to memorize... Whenever possible, teachers should use guided inquiry and interactive activities so that students can discover concepts themselves instead of simply having the concepts told to them.”

Scott’s drawing of his own experience as a learner (see Figure 5.1) showed students sitting in groups of three, but facing a lone board where the teacher is lecturing. No peer interaction was indicated, nor was there any mention of dialogue occurring between the instructor and the students. Rather, “The teacher is illustrating trigonometry using a diagram and students are taking notes;” an instructional approach that could be referred to as the banking concept of education (Freire, 2005). In Scott’s ideal classroom, however, students would be seated in pairs and manipulatives would be placed on the table to facilitate peer-to-peer interaction and knowledge construction through Discourse (Gee, 1999). These actions would corroborate Scott’s belief that “Mathematics is a fluid subject wherein you can be creative and discover things for yourself,” as indicated on the pre- and post-semester surveys (see Table 5.1).

Figure 5.1



In his Ideal Classroom illustration, Scott drew himself circulating the room and assisting students as needed, rather than fixed at the front of the classroom as he experienced as a learner. There would be multiple boards, displaying various guides for students for the day's lesson and beyond, and as marked on his surveys, Scott would potentially include language objectives and instructional strategies as a part of his practice. SMART Boards would be utilized as assistive technology for ELLs as well as more visual and kinesthetic learners, again, increasing their opportunities for Discourse (Gee , 1999). In addition, he also would provide options for pupils who operate more quickly, maintaining "an extra credit problem for students who finish early" on one of the boards.

“... I do not want students to be taught mathematics
in the same manner in which I was taught....”

Mabel was a Caucasian female who was raised and was still residing in a town across the river from a large urban city. Her mother was a teacher and was a district administrator in the school system that encompasses this town at the time this research was being conducted. Mabel attended a state university for her undergraduate degree in History and Mathematics, and chose the institution of this study for her graduate degree because “they gave [her] the most money.” She was a part of an urban teaching cohort within a Master’s degree program and desired to establish her career in an urban school district. As to whether she would return to teach in the town in which she grew up, which has only one public high school, Mabel deliberated,

“It depends ‘cause my mother is in [my hometown], so that makes it a little more difficult. Although, like, I haven’t had bad experiences in [my hometown], it’s just there’s always tension with some people. So I think, you know, it’s something that I, I’ll just have to decide a year from now what I want to do. Look at my options, and decide from there.”

According to her pedagogical philosophy, Mabel desired to engage her future pupils in mathematical learning through means such as exploration and discovery, and cooperative learning, although she wrestled with this aspect of mathematics in her pre-semester survey (see Question 14 in Table 5.2) and interview. In the latter she stated,

“There are some aspects that, I feel like might just go over a student’s head. It’s just not as easy to discover it. I know even when I was learning math, I needed a

professor to be like, ‘Hey! This is how you do it.’ So, I think there is a level of discovery, but I also think there is a level of you need to be taught certain things.”

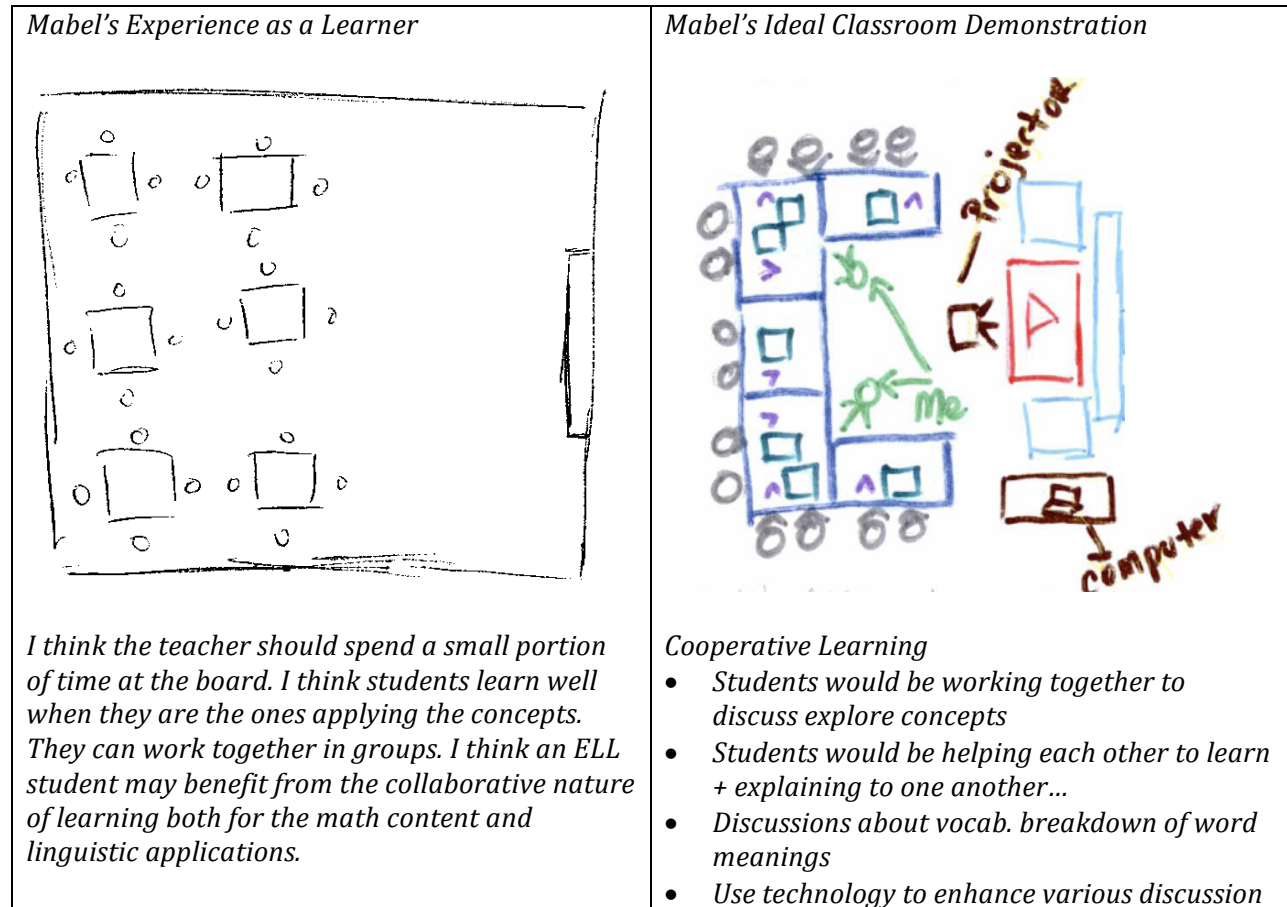
By the end of the semester, she strongly agreed with the more fluid and creative nature of mathematics and planned to establish a “safe” learning environment, where students’ “ideas are valued and confidence in their mathematical abilities is fostered throughout their learning,” as indicated in her post-semester survey’s Ideal Classroom Demonstration (see Figure 5.2). In her drawing, she displayed her students seated in and working in groups, while she is circulating the classroom, “providing feedback and offering guidance.”

While Mabel also presented groups for her personal experience as a learner, she did not offer as much detail as to how these groups would function nor the tools that she would utilize to enhance student learning. Mabel’s illustration indicated that she may not have read the prompt for the drawings very carefully as she states a few times what she thought should occur in a mathematics classroom, as opposed to what she actually experienced.

Table 5.2. Mabel’s Responses to Questions of Interest

Survey Item	Theme	Mean (pre, post)
Q12. I have adequate training to work with ELL students in a secondary mathematics classroom.	Preparedness	1, 2.5
Q14. Mathematics is a fluid subject wherein you can be creative and discover things for yourself.	Mathematics philosophy	2.5, 4
Q17. Teachers should not modify their instruction for the ELL students enrolled in mainstream mathematics classes.	Modification knowledge	3, 2
Q22. Language development should only be taught within English courses.	Mathematics teacher’s role	2, 1
Q23. As a mathematics teacher, I intend to concentrate exclusively on mathematics-content objectives for my classes.	Preparedness	2, 2

Figure 5.2



Mabel began the fall 2010 semester with three teacher-education courses completed from the summer, and she intended to enroll in a bilingual education course in the spring 2011 semester. Prior to beginning her studies at the university, her teaching experience was comprised of tutoring high-school juniors and seniors who had not passed the state graduation exam and serving as an interim teacher in the Geometry Learning Center at her former high school, where she found herself poorly equipped for such a task: "That was a very challenging time... made me want to teach math though." Mabel ended the semester feeling moderately trained to work with ELLs in mainstream mathematics classrooms and

moderately confident in her ability to instruct her students in the language of mathematics, according to her response to Question 12 in the post-semester survey (see Table 5.2).

While she remained indecisive about the point in which ELLs should enter mainstream classes, Mabel came to agree with the use of students' native language as a classroom tool, a feature of "skilled" teaching (Torres-Velasquez & Lobo, 2004/5), as well as with the modification of instructional methods in order to provide ELLs better access to the content, which she indicated on Question 17. Her disagreement with providing modifications came from an interpretation she had received in a prior class:

Mabel: I was taught in my diverse learners class that accommodations are more of, you know, you still have the same standards, but you sort of just help the students – like, it's not actually changing the standards and not changing the curriculum.

Where that's modification; modification is you actually make those changes.

Interviewer: So you change what is even expected of the student?

Mabel: Yeah.

Once she was assured that this was not the intention of the question, but rather that *modify* had to do with putting strategies in place that would provide students access to the content, Mabel verbally changed her selection to "disagree." According to Questions 22 and 23, respectively, Mabel ended the semester with a more fervent belief in language instruction across content areas and confirmed her desire to include both language and content objectives in her instruction. "There's language in math!" as she noted on her survey. Finally, Mabel also felt more assured of her ability to determine appropriate content objectives by the completion of the course, which she doubted in her pre-semester survey and interview.

“...it’s absolutely critical and necessary that we be mathematically proficient...”

Bryce was a Chinese-American male who attended a coeducational Jesuit high school in the Midwestern region of the United States before enrolling at this northeastern college. His decision to attend this university was accidental:

“I came to [the university] – it was kind of unexpectantly. I was, I went to a Jesuit high school and heard about [the university] many times. And I came to [this urban area] to visit some other schools, and I just happen-chance decided to go to [the university] ‘cause I had some extra, a couple of hours to kill, to visit. And I fell in love with it. And then I applied, and the rest is history....”

He was a senior who began his studies at the university as a mathematics major, but upon taking a few education courses, decided to expand his degree to a double major in secondary mathematics and mathematics from his sophomore year. He also received admission to the fifth-year master’s degree program at the college, but was planning to complete it on a part-time basis. Bryce would be the first person in his family to pursue a teaching career, a decision that had not been the most agreeable with his family:

“Actually they were not really happy that I was going to be a teacher. They were disappointed that I wasn’t going to be either a doctor, Fortune 500 CEO so... Yeah, so they were somewhat disappointed...”

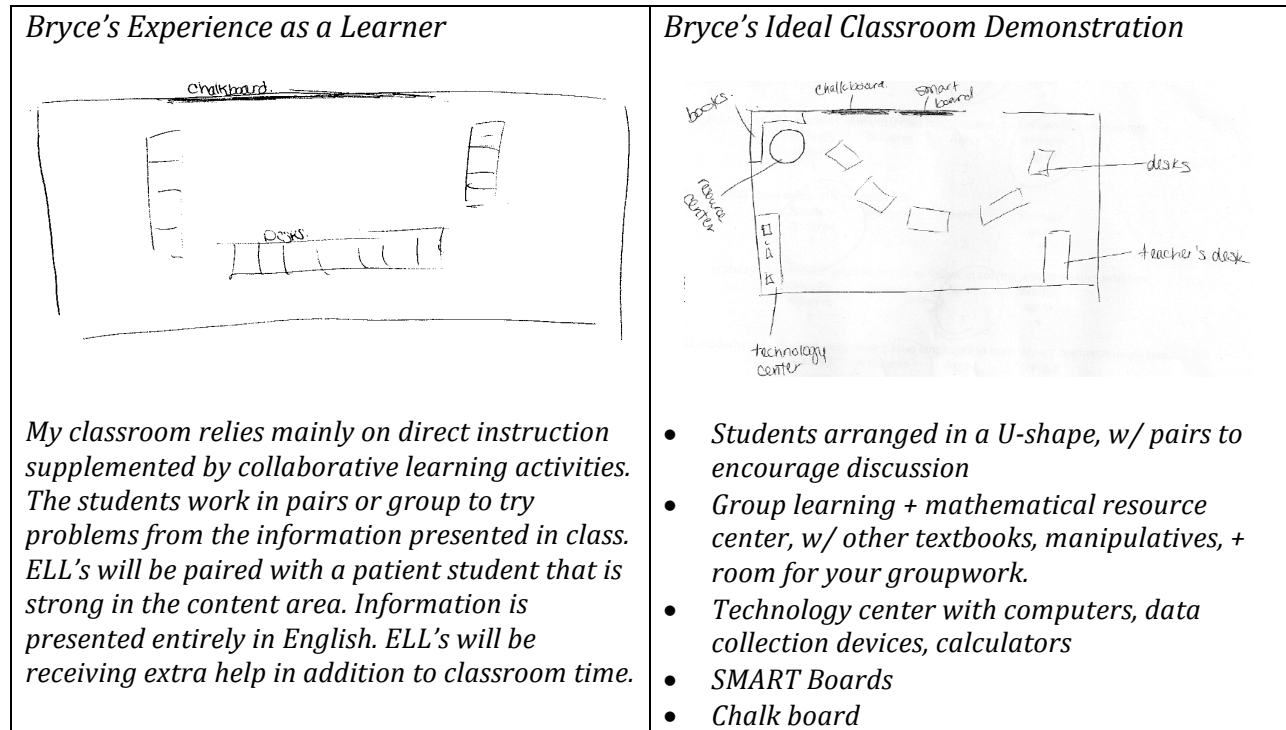
Considering himself a disciple of the instructional paradigm set forth by Saint Ignatius of Loyola, Bryce described his pedagogical philosophy as consisting of three key ideals: contextual learning, variety of instructional methods, and high standards for all students. He believed in the coexistence of mathematical theory and application equally in order for “students to gain a true appreciation of mathematics.” Bryce expressed

acceptance of both individual and collaborative learning, and saw himself as “a guide for students to discover mathematics that they will constantly build upon as a lifelong learner.” In maintaining high standards for all his students, Bryce intended to present the content in a way that would be “rigorous and challenging,” and he planned to emphasize mathematical literacy for all students. It is critical that all three of these intentions coexist, as Beykont (2002) asserted,

“Excellent teaching across language difference in mainstream classrooms requires the moral resolve to hold all students to high academic standards, the political clarity to understand the sociohistorical factors that have inhibited language minority student success, and the pedagogical expertise to use the power of culture as a means of offering the educational supports language students need to succeed in U.S. schools” (p. xxx).

In Figure 5.3, Bryce’s drawings depicted U-shaped configurations for the students’ desks both in his own experience and in his ideal class. However, the latter illustration separated the desks more; yet his narrative described students being seated in pairs in order to facilitate groupwork. Interestingly, in his post-semester interview Bryce seemed somewhat apprehensive about the affect of collaboration onto the learning process: “I want them to get it down themselves first, and then be able to carry something to the group and be able to work together, rather than having them use the group as a crutch.” Bryce mentioned in the narrative for his experiential drawing that he would desire some collaboration as well, especially for the support of his ELLs, but in a supplemental capacity rather than as a primary learning activity.

Figure 5.3



Despite this, Bryce designated a group learning area as well as other learning centers in his ideal classroom. The “books” area would consist of a variety of resources in order to assist Bryce in addressing the diverse needs of his students. The “resource center” would be an area he dedicated to one-on-one tutoring. The “technology center” would be especially for statistics work with computers, calculators, and other aids. Ideally, each student would have access to these tools, but at worst, Bryce envisioned his pupils utilizing technology in pairs or small groups. None of these centers, tools, and resources were present in his experiential drawing.

While the teacher’s desk in his ideal classroom would be located in the back of room, signifying a more student-centered instructional approach, Bryce noted during his post-interview his desire for students’ focus to lie towards him during the learning process: “I always liked that U shape, because I like to have a center stage, but I also like it, because it

feels, it's more of a discussion feel, because I love it when you can just sit there." This statement ran a bit contradictory to the philosophy he penned earlier in the semester: "Both group and individual learning experiences are incorporated into a comprehensive collaborative learning process that includes all students as well as the teacher. In my classroom, I do not view myself as imparting knowledge to the students; instead, I am a guide for the students to discover mathematics that they will constantly build upon as a lifelong learner." In that Bryce also noted a firm belief in the creative and exploratory nature of mathematics (see Question 14 of Table 5.3), it was unclear as to how he intended to promote this view and yet be the center of attention in his future classroom.

Table 5.3. Bryce's Responses to Questions of Interest

Survey Item	Theme	Mean (pre, post)
Q12. I have adequate training to work with ELL students in a secondary mathematics classroom.	Preparedness	2, 4
Q14. Mathematics is a fluid subject wherein you can be creative and discover things for yourself.	Mathematics philosophy	3, 4
Q17. Teachers should not modify their instruction for the ELL students enrolled in mainstream mathematics classes.	Modification knowledge	1, 1
Q22. Language development should only be taught within English courses.	Mathematics teacher's role	4, 2
Q23. As a mathematics teacher, I intend to concentrate exclusively on mathematics-content objectives for my classes.	Preparedness	3, 2

In spite of the vagueness of Bryce's sense of the usefulness of collaboration, Bryce did reveal in the questions reported in Table 5.3 his feelings of preparedness to address the needs of linguistically diverse students in his classroom to come (Question 12), his belief in the employment of modifications to assist this students (Question 17), and his intentions to consider more than just content objectives (Question 23). While he also indicated

agreement with the limiting of language development to English courses, Bryce retracted his response when questioned during his post-semester interview, stating, “No, I mean, language development is in any course.”

Bryce was fluent in Chinese and has some conversational fluency in Spanish. He visited Spain twice on service-oriented excursions and credits these experiences for developing his sympathy and empathy for ELLs.

“I mean language was just really a huge barrier, and, I mean, I had speaking Spanish before but with other people on my trip, it was also much more difficult. And it’s just really difficult to communicate in a language that you’re not, your brain is not hardwired to communicate with.... I think the biggest thing for me was vocabulary. It was like I knew how to conjugate verbs and I knew how to properly use what tense... I would say, I was like making up some words along the way. And they were like looking at me, like, ‘I don’t think that is an actual word.’ I mean it’s a, it’s very difficult to not have the language of the area that you are speaking around you.”

Prior to the methods course, he only partook in the two-hour academic-language trainings offered by the practicum office at the beginning of each term. As the fall 2010 semester marked his third pre-practicum placement, he had participated in the trainings thrice. He had intentions to take the bilingualism and literacy course at some point during his graduate program and would like to pursue the certification for teaching ELLs, if possible. Contrarily, Bryce’s Ideal Classroom drawing (see Figure 5.3) offered no indication of his intention to address the needs of linguistically diverse students in the future.

“... all the girls were sent there by the State...

that probably made me want to teach more than anything...”

Natalie was a junior at the university who transferred in her sophomore year from a southeastern public university. She originated from a small, farm town in upstate New York, but had led a varied life thus far. She attended a private Catholic high school, through which she traveled to a number of countries during her summers – Switzerland, Greece, and China, among others. Her mother was Italian, and her father was Scottish, but she claimed that Italian is her family’s second language.

Natalie took classes for three different languages between grade school, high school, and college – French, Latin, and Italian, respectively. However, she did not claim fluency in them. Her French lessons were “ten years where I never learned anything,” and she maintained, “Everyone but me and one other sister speaks fluent Italian.” She explained that she could understand what is being said, but could not speak the languages herself.

Natalie would be the first in her family to pursue a career in teaching, a childhood dream, much to the chagrin of her family:

“My mom says I’ve always wanted to be a math teacher... They all think I’m crazy.

My mother tries to get me to switch my major daily. Every day I talk to her, ‘Are you still doing that whole math teaching thing? Really honey? Are you sure you don’t want to switch?’ ‘No, Ma. No.’”

She was the youngest of five siblings; all of who chose other careers such as business, massage therapy, and journalism. Interestingly, Natalie did not feel that she was very “good at math” and credited having more “good” teachers for channeling her towards teaching the subject:

“I have a really hard time in it. But, like, I like it, kind of. It’s a hate, love/hate relationship. Like, when I see certain things, I’m, like, oh, I love that. But, I mean, like, I dread doing my homework in my math.... I guess it was a ratio of good teachers to bad teachers that made me realize how much it affected how much I actually cared about a subject. I mean, I used to hate math, because I had a horrible teacher for two years. And I would just sit in the back of the classroom, reading magazines, sitting on top of a desk. I was, like, the rebel. And then I had the teachers that would actually explain things to me. And I was, like, oh, like, I can do that.”

Natalie had two semesters of pre-practicum experiences prior to the fall 2010 semester. Her first experience was through a southern university from which she transferred, where she worked at the state’s school for girls who had “been in trouble with the law” and “were required to attending until sentencing.” Natalie described this experience as “intense,” but also worthwhile:

“My parents tried to pull me out of the school, because they didn’t want me teaching at that school. I was, like, ‘Yeah, there’s security guards, like, every ten feet’... But, yeah. I mean, it was the people who need it the most that make it most worth the while, so. That was great.”

At this school she worked with girls who were at different grade levels but had not successfully passed their mathematics courses. Her second semester of pre-practicum was at a diverse urban high school near the university in this study. At this high school, Natalie worked with a ninth-grade special needs class, as well as tenth and twelfth-grade classes. For both classes, most of the students were ELLs, for whose instruction Natalie had

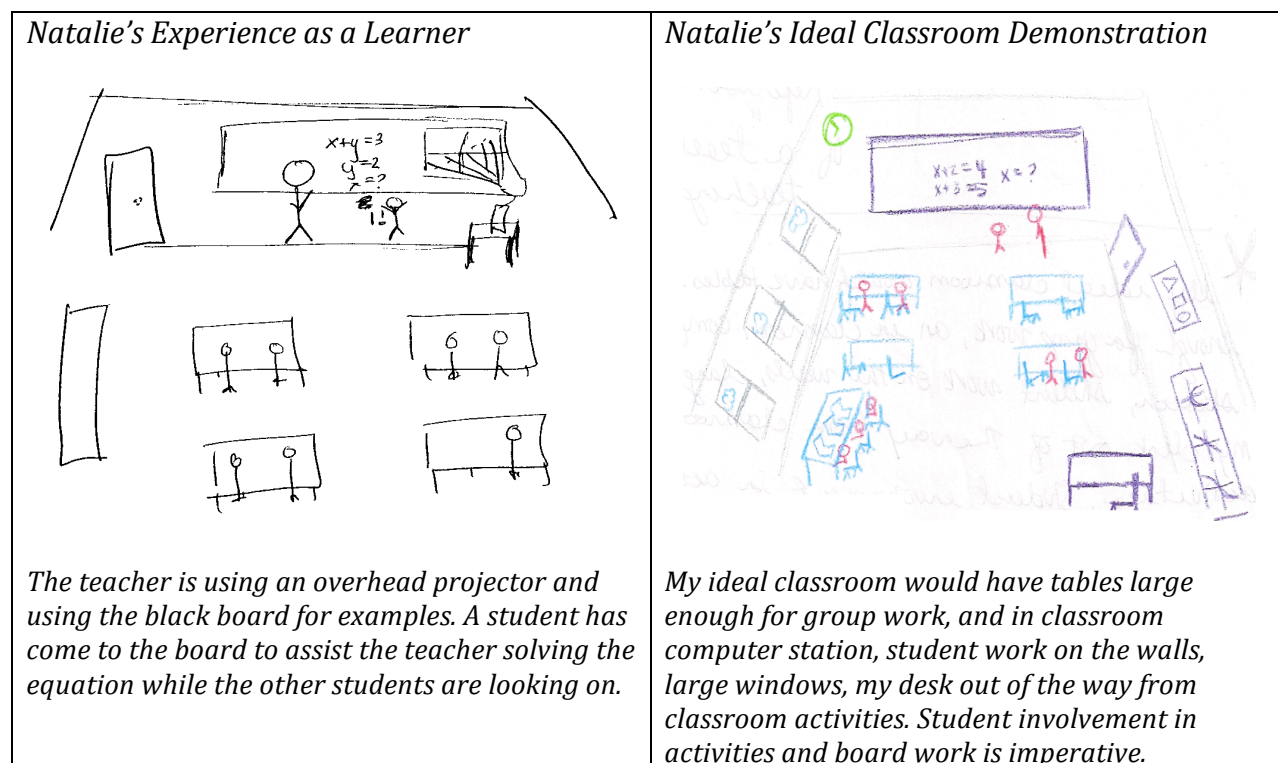
received no training. However, she credited her own travels, particularly to Switzerland, for making her sympathetic to the needs of non-native language speakers:

“... I did not want to speak in class. Everyone spoke French. And I didn’t want to talk. I was embarrassed and I would just pray that the teacher would not call on me... I was the only American, only English speaker.... I mean, that was really difficult. The teacher just, I was just kind of one in the crowd there. So the teacher didn’t really have much difficulty dealing with me, because she just didn’t.”

Natalie viewed mathematics as its own unique language, purporting that “if a sufficient base is not built, and continually reviewed and built upon, that any student will find the high level courses extremely difficult.” It was with this view that she outlined the first component of her pedagogical philosophy to be “reinforcement of the basics.” In addition, Natalie considered collaborative learning to be foundational to her classroom practice, as explicated in the following excerpt from her Case Study Analysis assignment and as corroborated in her Ideal Classroom Demonstration (see Figure 5.4):

“Most students have the ability to listen to what is said in lecture, or read a textbook, and then restate that same thing in class to their teacher. But for a student to be able to explain their understanding to another student, or even more so for a student to be able to help another student understand, is in a language that they are not only more comfortable with, but one that will stick with them.”

Figure 5.4



It should be highlighted that while her experiential drawing also featured students sitting in pairs, there was nothing drawn or highlighted to indicate that students would be required to function in a collaborative capacity in Natalie's classroom. On the contrary, she drew herself at the board with a student; however, she did illustrate groups of students engaging in other activities such as computer work while she was working with the single pupil. This may have alluded to the intended presence of differentiation in her future classes as well, or even the language modifications which she supported in Question 17 of the pre- and post-semester surveys (see Table 5.4).

Table 5.4. *Natalie's Responses to Questions of Interest*

Survey Item	Theme	Mean (pre, post)
Q12. I have adequate training to work with ELL students in a secondary mathematics classroom.	Preparedness	1, 3

Table 5.4. (*continued*)

Survey Item	Theme	Mean (pre, post)
Q14. Mathematics is a fluid subject wherein you can be creative and discover things for yourself.	Mathematics philosophy	4, 3
Q17. Teachers should not modify their instruction for the ELL students enrolled in mainstream mathematics classes.	Modification knowledge	2, 2
Q22. Language development should only be taught within English courses.	Mathematics teacher's role	1, 1
Q23. As a mathematics teacher, I intend to concentrate exclusively on mathematics-content objectives for my classes.	Preparedness	2, 2

In addition to group work, Natalie planned to implement a “balance of lecture, group work, individual work, and student presentations.” She believed in active participation from all in her classes and desires for her pupils to be engaged through a variety of means, as well as feel comfortable and free to be creative – all supported by the drawing and narrative for her ideal classroom (Figure 5.4), and also her belief about the nature of mathematics noted in Question 14 of the pre- and post-surveys (Table 5.4). The last key element of Natalie’s pedagogical philosophy was that of the integration of real-world application, and even experiences, into her classes. She stated, “There is no way that students are going to be able to find true interests in math if they believe that the work is only applicable within the classroom or in a textbook.” Thus, Natalie intended to form strong relationships with her pupils in order to design experiences that relate “to the material of [her students’] lives.”

In Table 5.4, Natalie indicated growth in her feelings about being prepared to teach ELLs (Question 12). She also believed that language instruction should not be confined to English teachers, but shared across all content areas (Question 22). Finally, she noted her

intent to include language objectives in addition to content objectives in her practice (Question 23). In her post-semester interview, Natalie was even able to assess the strength of language objectives she generated in for her group's Concept Presentation at the end of the course:

NATALIE: Oh, I think they're totally weak.

INTERVIEWER: What's weak about them?

NATALIE: They're very... I'm so blind. "Define an exponential function." Okay, so, I don't know, I feel like, even *there* would have been better than "define," defining the identities, describing them, then giving like different parts of the exponential function, not just, "What does exponential mean?"

INTERVIEWER: You get it.

NATALIE: "Explain why an exponential function grows more rapidly than a linear function." I think that's a good start to one. I think it could have gone more like into examples and into like, "Is there a time when a linear function does grow more rapidly than an exponential function?" ...

INTERVIEWER: Well, if you can look at them and see the weaknesses in them, that's a good sign. That means that you internalized some of the things we had been talking about as far as language objectives. So that's good. That's good. That means you have something to take away with you.

NATALIE: I hope so.

"I feel I have a responsibility to give back to the community..."

Hank was a native of the Dominican Republic, having immigrated to the United States at the age of 15. In his country of origin, he was a senior in high school, but upon

entering school in the U.S., he was relegated to the tenth grade, as, per Massachusetts' laws, his age required that he matriculate in a grade school. Hank had not learned any English prior to his arrival in the U.S. and thus, he participated in the Spanish bilingual education program at an urban high school during his first year in the U.S. and gradually moved into all mainstream classes by his second senior year.

Hank attended the university in this research for his undergraduate degree as well, yet he maintained his relationship with the urban high school that he attended. Upon his graduation from college, he became a mathematics teacher within the same bilingual program in which he had begun his studies in the U.S., as well as a Spanish and a sheltered English instructor. These experiences may have been the reason for Hank's feelings of being adequately trained to teach ELLs at the beginning of this semester, as noted in Question 12 of the pre-survey (see Table 5.5), but also the justification for the lack of change in his response to this prompt on the post-survey. Hank's teaching experiences prior to the methods course may also have accounted for his staunch support of modifications for ELLs in mathematics (Question 17), his embrace of language instruction beyond English courses (Question 22), and his desire to include more than just content objectives in his future practice (Question 23).

Table 5.5. *Hank's Responses to Questions of Interest*

Survey Item	Theme	Mean (pre, post)
Q12. I have adequate training to work with ELL students in a secondary mathematics classroom.	Preparedness	3, 3
Q14. Mathematics is a fluid subject wherein you can be creative and discover things for yourself.	Mathematics philosophy	3, 4
Q17. Teachers should not modify their instruction for the ELL students enrolled in mainstream mathematics classes.	Modification knowledge	1, 1

Table 5.5. (*continued*)

Survey Item	Theme	Mean (pre, post)
Q22. Language development should only be taught within English courses.	Mathematics teacher's role	2, 1
Q23. As a mathematics teacher, I intend to concentrate exclusively on mathematics-content objectives for my classes.	Preparedness	1, 2

His former ties to this high school eventually led him to a second teaching position at a pilot school within the same urban school district – the one that he was holding at the time of this study.

“... when I was a student at [the high school], there was a gentleman called [C, Mr. C]. Then when I went back to teach, he was still there my first year. But then he left to the [pilot school] that started in '99. And he kept telling me, “Oh, you should come with us. Come with us.” But I, I was really happy at [the urban high school]. I liked the kids. There was, we had a lot of newcomers, and I really enjoyed working with that population. And, but then when Question 2 was implemented, I decided to make the change. So I started working at the [pilot school].”

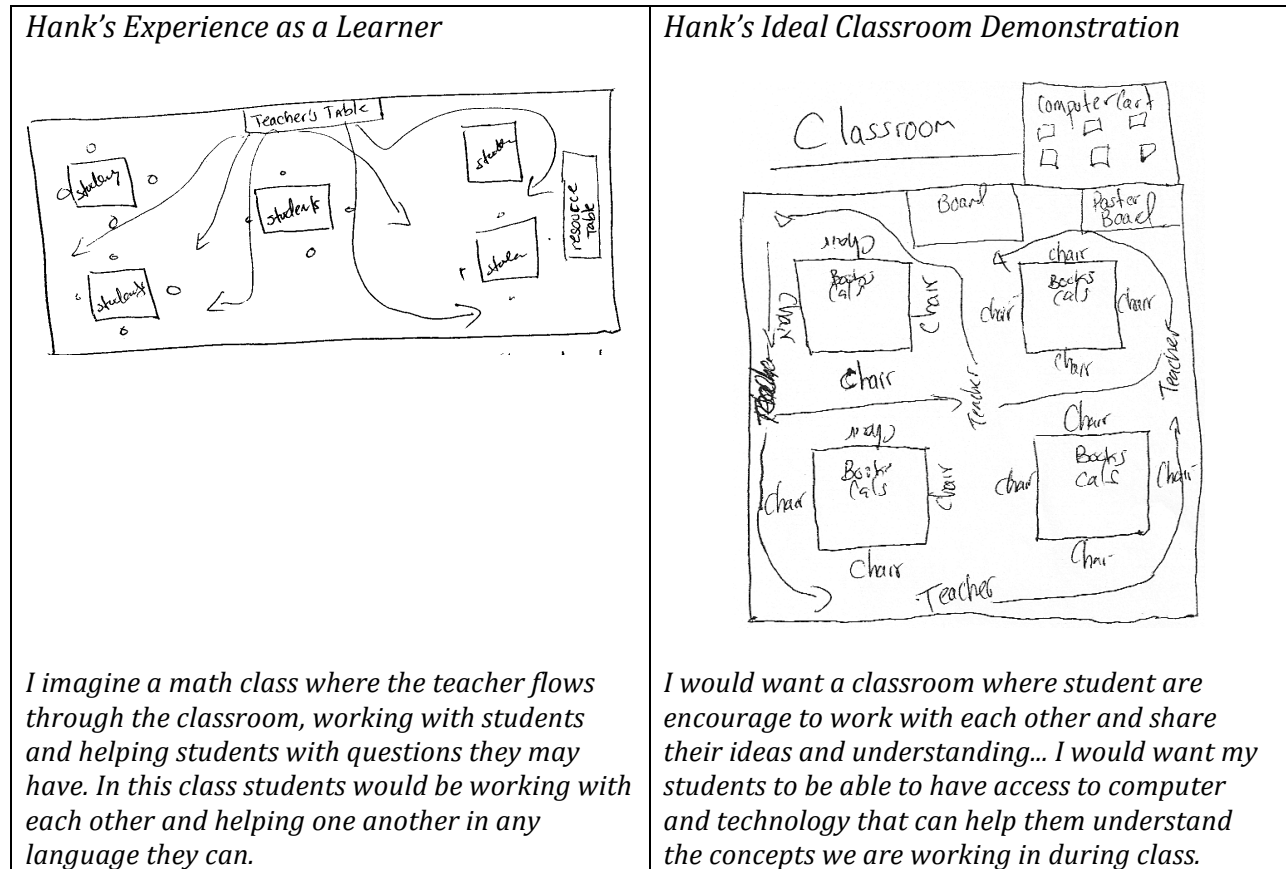
At the pilot school, Hank taught Geometry and two beginning Spanish courses, in addition to serving as a school liaison to Spanish-speaking families. His goal as an instructor was to “promote and develop a safe environment and fervent learning community.” To create such an atmosphere, Hank planned to build strong relationships with his students, differentiate instruction to accommodate their needs, and integrate new skills in his practice that would engage the students and provide “a positive and dynamic learning environment.”

Hank's mathematics classroom at the time of this study was shared with two other teachers, and so his ideal classroom (see Figure 5.5) was quite different from his reality.

“... it’s [his current classroom] kind of this horseshoe shape, where we have students around. We do have an overhead projector that our school got for every classroom this year. But there’s no calculators. There’s not like a place to put, you know, we have our textbook. There isn’t a place for, we don’t have computers. So yeah, that would be ideal for us to have calculators and a computer cart that we can kind of work behind and say, you know, ‘Let’s go here,’ or ‘Look at the page that I’m put...’ You know, it’s not, yeah, we’re not at that. Unfortunately.”

Ideally, Hank would not only be able integrate more technology into his practice, but also utilize a more cooperative learning style of instruction. He believed that the lecture approach to teaching very ineffective in middle and secondary grades, preferring instructional styles that lent themselves to the more creative and exploratory nature of mathematics (Question 14, Table 5.5). Thus, Hank’s role would be to introduce a concept and then allow his students to work together, “talking and relating ideas.” His ideal classroom diagram illustrated his movement around the classroom, assisting groups of students as needed. Hank’s experiential drawing also showed students in groups with the teacher circulating and assisting students. However, his narrative for the latter illustration described Hank’s ideal classroom, rather than his own history as a mathematics learner.

Figure 5.5



Interpretive Summary

The purpose of this chapter was to provide rich descriptions of the case-study participants and their teaching philosophies, as well as to begin discovering evidence of the content-and-language integrated learning intervention's influence on the participants' instructional approaches. The preservice teachers' drawings, assignments, surveys, and interviews were utilized in the creation of these portraits in order to give the teachers voice, "letting [the] research participants speak for themselves, either in text form or through plays, forums, 'town meetings,' other oral and performance-oriented media or communication forms designed by research participants themselves" (Guba & Lincoln, 2008, p. 278). In this study, the researcher and the course instructor generated most of the

design, but the opinions and philosophies expressed within the various media were that of the preservice teachers.

Gleaning from the participants' portraits, one might have expected to find LRP-related evidence across all five participants. Demographically, both Hank and Bryce were considered ELLs, plus Bryce had a short amount of experience as a language learner in another country. Natalie also had experience as a language learner, and although Scott, Natalie, and Mabel were classified as Caucasian, all three of them had histories of tutoring or teaching ELLs prior to this secondary mathematics course and the intervention. Based on the participants' backgrounds, Fillmore and Snow's (2002) concerns about teachers not being able to relate to the cultural and linguistic experiences had the chance to be allayed.

The participants' responses to the select survey questions and oral corrections to a few questionable choices confirmed their beliefs in LRP. They all ended the term with a stronger belief in the discovery-oriented nature of mathematics, and thus the need to provide such experiences in their classrooms. In addition, the preservice teachers noted the existence of language development beyond English classes, the intention generate both content and language objectives for their lessons, and the goal of modifying instruction for ELLs in their courses. Mabel's post-semester survey even pointed to the need for continued training in relation to these and other aspects of LRP. The hope is that the material culture examined in the Chapter Six will continue to corroborate the philosophical stances expressed by the students in this chapter.

Bryce and Mabel's drawings from the end of the semester indicated possible deterrence from the linguistically response approach, however. Albert (in press) posits, "Teacher-generated drawings and narratives can serve as internal communicative tools' for

mediating inner thoughts about mathematics teaching and learning, as well as tools that facilitate external communications, with others” (p. 5-6). Both Bryce and Mabel’s illustrations show U-shaped arrangements for their students’ desks with teacher as the “center of attention,” as Bryce phrased it. Nevertheless, Mabel detailed her students’ experiences as being based in exploration in pairs with students “helping each other to learn + explaining to one another;” whereas Bryce allotted a group learning “center” in his classroom.

The other three participants drew pictures for their ideal classrooms that seated students in pairs or in small groups as well. Hank’s drawing had his pupils seated in groups of four with arrows drawn from the teacher all over it, indicating the movement of the teacher around the room and alluding to the teacher’s function as more of a facilitator. Interestingly, both Natalie and Scott’s students were paired and facing the front of the room, besides the small group working on computers in Natalie’s drawing. Scott’s illustration had the teacher moved away from the front board, whereas Natalie’s picture had her placed at the front of the board. These visuals were worth some attention, because while students can be grouped, it does not necessarily follow that they will function as collaborative teams. Lucas and her peers (2008) counsel, “The focus and nature of the interaction are also important” (p. 369), as students needs to given tasks that “involve the negotiation of meaning, not carrying out an exercise that requires little thought.” Scott, and also Mabel, showed various materials on the students’ desks, indicating the implementation of more activity-based lessons wherein students would be required to work together. Hank noted the presences of books and calculators on the desks, which could be used for engaging tasks, but special attention would need to be paid to the design of the lesson.

Haney and his peers (2004) cited student drawings as “a powerful vehicle for teachers to learn from students’ perspectives” (p. 244). Unfortunately, because three of the five preservice teachers drew pictures of a more ideal classroom for their personal experience illustrations, the impact of the content-and-language integrated intervention on the final drawing could not be assessed. It was unclear as to whether the participants felt rushed to complete the survey and, therefore, did not read the drawing prompts correctly or if they did not understand the directions. What was distinct across the end-of-semester illustrations through a holistic lens, however, was the importance of social interactions in the learning process. All of the preservice teachers illustrated or described the seating of their students to be in small groups or pairs in their ideal classrooms. In addition, computer stations or carts were in every ideal drawing and SMART Boards were in two of them, indicating a propensity for the use of technology beyond calculators in the participants’ classrooms. The remaining material culture, analyzed in Chapter Six, will have to serve as a better evaluation of the impact of the intervention.

Chapter 6

QUALITATIVE RESULTS & ANALYSIS:

EVIDENCE OF LINGUISTICALLY RESPONSIVE INSTRUCTION

This chapter will serve as an evidentiary examination of the multiple-case study participants' material culture produced in the methods course (Critical Analysis, Case Study Analysis, Imagine Ifs, Lesson Reflection and Organizer, and the Concept Presentation), as well as the Academic Language Projects submitted to the university's Practicum Office at the end of the same semester. The results from the Concept Presentations, in particular, will be of interest, as this was the first in-class opportunity to observe content instruction by the course enrollees that could potentially include strategies to address the needs of linguistically diverse students without any prompting from the methods instructor in the rubric for the assignment. From this analysis, conclusions about the effectiveness of the content-and-language integrated intervention will be drawn, in order to respond to the overarching research question and the second supporting question:

How does a content-and-language integration intervention effect the preparation of secondary mathematics preservice teachers in order to improve instruction for diverse learners, more specifically ELLs, in mainstream mathematics classes?

- b. Do the preservice teachers' ability to recognize academic-language challenges for ELLs evolve during the secondary mathematics methods course? If so, how?

Beykont (2002) argues, “it is unrealistic to expect language minorities to succeed with no extra supports in an educational system that was designed for a homogeneous, native English-speaking student population” (p. x). Therefore, as each artifact was examined, the intent was to identify instances wherein each participant noted a pedagogical practice or strategy that would assist in students’ language acquisition through their educative experiences in mathematics and which may have been taught during the methods course. These occurrences were coded, and then, in order to further dissect the findings, codes that were applicable to the conceptual framework were extracted. Tables 1, 2, and 3 in Appendix G show the results of this analysis. These codes were next sorted into categories whose labels developed according to the codes’ relations to one another. Table 6.1 shows the results of this effort. The frequency with which each participant demonstrated knowledge of each conceptual component was then retallied according to the emergent categories from this table. These results are shown in Tables 6.2, 6.3, and 6.4. Each code reference was filtered to discount quoting or restating of an author’s work. Any instances noted in the category tables were the participants’ thoughts, actions, or observations. For instance, Bryce stated in his Critical Analysis assignment, “the researchers also suggest that oral communication has less of an enduring quality than electronic communication, and traditional teachers are less likely to communicate with their students because of the constructivist nature of their classrooms and that their students were more ‘college-oriented,’ thus requiring less student-teacher interaction.” However, he was rephrasing the researcher’s work, this kind of citation was removed from the final counts for this code.

Table 6.1. *Emergent Categories for Describing the Components of the Conceptual Framework*

Conceptual Framework Component	Categories	Codes Identified
Comprehensible input	Use of everyday language	1. Everyday language to define math terms 2. Everyday words with different meanings in math
	Language instruction & accommodations	3. Language instruction in math 4. Language objectives 5. Possible language accommodation 6. Scaffolding language 7. Specific tools for language instruction 8. Symbolic language focus 9. Vocabulary focus
	Differentiation considerations	10. Multiple intelligences 11. Multiple representations 12. Scaffolding content 13. Variety in instructional methods 14. Variety in pacing
	Making connections	15. Placement of activity within unit 16. Prior knowledge of students 17. Real-world connections
Student engagement	Student actions	1. Communication emphasis 2. Evidence of student engagement 3. Hands-on investigation or manipulatives
	Collaborative interactions	4. Cooperative learning 5. Heterogeneous grouping 6. Pitfalls of cooperative learning
	Teacher assistance	7. Questioning technique 8. Talk moves 9. Teacher as facilitator
	Student disposition	10. Role of student effort 11. Role of student ownership
Comprehensible output	n/a	1. Hypothesis testing function 2. Metalinguistic function 3. Noticing-triggering function

Table 6.2. *Frequency of Comprehensible Input Categories per Artifact*

Categories	Artifacts				Lesson Organizer & Reflection	Academic Language Project
	Critical Analysis	Case Study Analysis	Imagine Ifs			
Use of everyday language		Natalie (1)			Bryce (2) Mabel (1) Scott (1)	Bryce (1) Mabel (7)
Language instruction & accommodations	Mabel (5) Natalie (1)	Bryce (7) Hank (1) Mabel (3) Natalie (1) Scott (1)	Bryce (5) Hank (5) Mabel (3) Natalie (1) Scott (4)		Bryce (8) Hank (1) Mabel (14) Natalie (5) Scott (1)	Bryce (60) Mabel (66)
Differentiation considerations	Bryce (2) Mabel (6) Natalie (1) Scott (1)	Bryce (5) Mabel (4) Natalie (5) Scott (2)	Bryce (10) Hank (3) Mabel (1) Natalie (4) Scott (4)		Bryce (4) Hank (1) Mabel (6) Natalie (1) Scott (7)	Bryce (5) Mabel (3)
Making connections	Hank (1) Mabel (1) Scott (3)	Bryce (1) Hank (1) Mabel (2) Natalie (4) Scott (5)	Hank (1) Mabel (3) Scott (1)		Bryce (1) Hank (2) Mabel (5) Scott (3)	Bryce (6) Mabel (13)

Table 6.3. *Frequency of Student Engagement Categories per Artifact*

Categories	Artifacts				Lesson Organizer & Reflection	Academic Language Project
	Critical Analysis	Case Study Analysis	Imagine Ifs			
Student actions	Bryce (1) Mabel (5)	Bryce (2) Hank (4) Mabel (3) Natalie (2) Scott (5)	Bryce (1) Hank (1) Mabel (2) Scott (2)		Bryce (2) Hank (5) Mabel (1) Natalie (4) Scott (5)	
Collaborative interactions	Bryce (2) Mabel (3) Natalie (1)	Bryce (1) Hank (4) Mabel (1) Natalie (4)	Bryce (3) Mabel (3) Natalie (2) Scott (2)		Bryce (5) Hank (2) Mabel (2) Natalie (5) Scott (2)	Bryce (4) Mabel (4)
Teacher assistance		Bryce (5) Hank (1) Mabel (4) Natalie (1) Scott (3)	Scott (2)		Bryce (3) Mabel (3) Natalie (1) Scott (5)	Bryce (11) Mabel (21)
Student disposition		Scott (2)	Hank (1) Natalie (2)			

Table 6.4. *Frequency of Comprehensible Output Categories*

Code	Artifacts				Academic Language Project
	Critical Analysis	Case Study Analysis	Imagine Ifs	Lesson Organizer & Reflection	
Hypothesis testing function	Mabel (1)	Bryce (1) Hank (1) Mabel (1) Natalie (2) Scott (5)		Bryce (1) Mabel (1) Natalie (2)	Bryce (4) Mabel (23)
Metalinguistic function		Hank (1) Natalie (2) Scott (2)		Bryce (2) Natalie (4) Scott (1)	Bryce (1) Mabel (3)
Noticing-triggering function		Scott (1)		Natalie (2) Scott (3)	Mabel (11)

Additionally, Bryce and Mabel are the only participants listed in the Academic Language Project data as they alone submitted their projects. Natalie changed majors towards the end of the semester; Scott did not complete the program; and Hank simply neglected to submit his project. Finally, the Imagine Ifs artifact in the tables includes all four of the exercises administered during the course of the semester, and the Academic Language Project consists of four lesson plans and a compilation of ten observational reflections (one reflection per week for ten weeks; roughly one or two paragraphs in length per reflection).

The next sections of the chapter discuss the results of the categorical tables in connection to the three conceptual framework components for students' language acquisition: comprehensive input, student engagement, and comprehensive output. An additional section will discuss the observations conducted during Concept Presentation, which was a group assignment. Finally, an interpretive summary will be offered through the lens of the conceptual framework in order to address the research question and to draw conclusions about the content-and-language intervention.

Results for Comprehensive Input

Echevarría and her peers (2004) define comprehensible input as, “making adjustments to speech so that the message to the student is understandable” (p. 66). Four categories of these “adjustments” emerged from the codes pertaining to the comprehensible input component of this study’s conceptual framework, as outlined in Table 6.1: *use of everyday language, language instruction and accommodations, differentiation considerations, and making connections*. Of these categories, the most prevalent among the participants’ artifacts was the group of codes for *language instruction and accommodations*.

Table 6.2 itemizes each grouping’s frequency within each course assignment for each participant. Interestingly, the assignments in the table are listed in the order in which they were assigned across the span of the methods course. Therefore, if the preservice teachers were being taught instructional strategies during the semester, then the desired frequency pattern for the purpose of demonstrating the effectiveness of the content-and-language integrated intervention would be for the number of occurrences to increase over time (i.e. from the first assignment to the last. The *language instruction and accommodations* category follows this desired pattern, with only six codes cited in the Critical Analysis assignment and 126 in the Academic Language Projects. Upon further scrutiny, the codes that comprise most of this category’s occurrences are *possible language accommodations* (59 times, as listed in Appendix G, Table 1) and *vocabulary focus* (63). The participants mentioned possible accommodations in a sporadic fashion within the first three artifact groups, but became much more intentional in their Lesson Organizer and Reflection assignment – Bryce (4), Mabel (6), and Natalie (4). Bryce and Mabel had even

more frequency in the Academic Language Projects; 12 and 17, respectively. As the Lesson Organizer and Reflection and Academic Language Project were assignments wherein the preservice teachers had to generate their own lessons rather than critiquing or observing others, the higher numbers for these two assignment may point to more opportunity to employ language accommodations when designing one's own lessons as opposed to critiquing others' lessons.

Possible Language Accommodations

The possible language accommodations that the participants mentioned are varied and some are stronger than others. With this in mind, the adjective "possible" became a deliberate and necessary choice in this code's name as some instructional devices listed by the participants possessed the potential to be actual language accommodations, but may not have been adequately described so as to make a more definitive determination. For example, in the third Imagine If exercise Bryce commented, "Supplementing questions with visual representations may also help students." Visual representations such as maps, tables, and illustrations can be very effective language accommodations, as they "can quickly convey considerable information to student, thereby reducing the amount of auditory information that they must process to make sense of the instructional topic" (Lucas et al., 2008, p. 368). However, no specifics were offered in Bryce's statement, making it appropriate to qualify Bryce's recommendation as "possible" assistance.

For other potential accommodations, Bryce highlighted the use of questioning techniques, metaphors, differentiation, and student explanations – discourse-based components – but also, graphical and tabular representations, technology, and algorithms. The latter elements expand Bryce's accommodations to embody Gee's (1999) Discourse

theory, where in communication employs more than verbal or written language, but also symbols, tools, and/or objects that may not utilize words. In the Lesson Organizer and Reflection, he further detailed,

“In the same way that asking students to perform several algorithmic calculations does not lead to deeper understanding, students must be asked to explain, both in speaking and writing, the process used to answer a problem, and technology can facilitate this learning process. In my lesson, I was able to ask students to explain the connection between graphs of the function to the Factor theorem, such as how the linear factors will appear as x-intercepts in a graph. For ELL students, this would have allowed them to connect visual representations to their understanding of the theorems.”

In his Academic Language Project Bryce also introduced a graphic organizer, which can offer students a concrete manner to organize their ideas in order to “clarify concepts, understand causal relationships, and trace the sequence of events” (Lucas et al., 2008, p. 368). Bryce’s organizer was designed to resemble a Facebook page (see Appendix H) – not only offering possible language assistance but also making a connection to his students’ very probable reality. Hank reiterated the use of visuals as potential support for language access in three of his four citations:

“One accommodation that I would use for ELLs and diverse learners is that I would provide more time and most importantly I would provide visual aids, for the students to know what they need to do. “ (Imagine If #2)

“I believe that one of the problems that English Language Learner’s may have in trying to complete this test is that understanding the language may be a problem.

For example, students may be able to answer the math questions if they were given visual representation of the questions and what they needed to do.” (Imagine If #2)

“I also believe that Eun Sil would benefit, as any student would, from a visual representation of what she is expected to do.” (Imagine If #4)

Like Bryce, Hank did not specify what these visuals would be (e.g. graphs, flow charts, etc.), but ties to the recognition aspect of Discourse (Gee, 1999), and thus, the potential for accommodating language, is present in his thinking.

In Hank’s fourth example of possible language accommodations, he mentioned student explanations as a potential tool: “As we have learned from class readings and discussions there is different level of understanding when you are able to explain something to others.” In contrast, Scott’s material culture was only found with one reference to a possible accommodation, situated in the second Imagine If exercise: “The first thing I notice is that questions 1-5 are very heavy on vocabulary. I would have rephrased question #3 and provided a diagram for question #5 in order to reduce confusion and assist ELLs.” Again, a visual representation is recommended, but no details submitted.

Mabel mentioned accommodations similar to the three male participants, but also adds to the list communication with fellow students, specifically in heterogeneous groups; displays of graphs; teacher modeling; and students writing definitions or notes in their own words. In her Critical Analysis, Mabel described the potential language access inherent to cooperative learning:

“Along with the multimodal approach is the cooperative learning approach, in which students are learning together. This approach can be beneficial in both the learning

of mathematics concepts, but also in the development of communications skills with peers.”

In the Lesson Organizer and Reflection, Mabel depicted how she would envision employing a visual representation (in this case, a graph) as a language accommodation:

“I think that the use of technology in this lesson will enhance the understanding of this vocabulary because it offers additional visualization of the words in action. For example, images of two coordinate grids will provide students with the opportunity to see a class visualization of two graphs that they had created, while the juxtaposition of the two graphs next to each other provides an easy compare and contrast moment for the class. In this instance, students understand the concept of noticing similarities and differences, which is a task that is asked of students beyond mathematics classes.”

Natalie’s only mention of possible language accommodations also fell within her Lesson Organizer and Reflection and centered around visual representations as well. In this excerpt, a class activity wherein the students created anchor charts in small groups was outlined:

“The activity that I settled on was having students divide into groups of four in order to create a poster about a given function. Each group was given a large piece of gridded poster paper with a large XY-plane already drawn on it, the function name at the top, and then spaces to fill in the blanks that read equation, characteristics, and examples. Students were able to use their notes and their graphing calculators to fill out the poster as completely as possible and to draw the base graph and examples of functions from that base graph on the XY-plane.”

While she never denoted the charts as a possible accommodation, she described them as helpful for students in the self-creation of the posters and for their future reference: “I think these activities will help the students remember at least their own function (from the posters), and they will certainly have the other posters on the walls to refer to in the future.”

Vocabulary Focus

The prominence of the *vocabulary focus* code was not surprising as many mathematics teachers tend to relate language use and instruction in mathematics to content-specific vocabulary. Interestingly, Natalie had no quotations highlighted under this code, perhaps indicating a perspective towards the register of mathematics that is not terminology-specific but rather more holistic. Hank and Scott each featured one related quotation for the *vocabulary focus* code:

“One other problem that I see ELLs having is that Geometry is a subject of shapes and dimensions, and students need to draw pictures in their heads about the requirement and then use that to solve the problem. The problem here is that ELLs may understand but may have difficulties translating their word knowledge into pictures that they can use to solve the problem.” (Hank, Imagine If #2)

“The word ‘coefficient’ could be confusing; I would remind them what a coefficient is and remind them what to do when you don’t see a number (e.g. x or $-y$).

The word ‘triangularize’ is terrible – it’s a noun changed into an adjective changed into a verb. I would tell them explicitly what they are trying for (zeroes in the three positions in the bottom left of the matrix) and then mention that it is called

‘triangularization’.” (Scott, Imagine If #4)

Hank raised a potential disconnect between content knowledge and comprehending the question being asked in the mathematics prompt. While Scott illuminated the complexity of the word “triangularize,” and subsequently suggested focusing on the computational process first, labeling the process with the official terminology afterwards.

The majority of the citations for the *vocabulary focus* code fell with Bryce, who totaled 29, and Mabel, who had 24. Regarding the terms and the problems that an ELL could potentially face with them within Imagine If #4, Bryce’s recommended, “If Eun Sil is struggling with vocabulary, perhaps providing translations of the vocabulary terms might be helpful, to scaffold initial learning. Eventually, she could also be moved to bilingual worksheets, etc.” He further commented, “Examples and demonstrations with consistent phrases and vocabulary will help her adapt her mathematical context with the current language.” The latter suggestion might benefit the computational ability of the student moreso than her capability to navigate the language involved in the problem. For the same exercise, Mabel offered a slightly different solution:

“Eun Sil needs instruction that emphasizes vocabulary more clearly. It could be beneficial to have the student break the problem apart, taking it one step at a time. Likewise, each step should be interpreted by the student in their own words and their own understanding from what was previously taught. I would also provide any clarification necessary. If the student is proficient in solving the problem without the matrix, they could look at a simpler problem and matrix and notice the similarities. Triangularization should also be explicitly explained so that the student understands the relationship of this term to the process they have been performing.”

Again, she emphasized students being allowed to express their thoughts in their own words and to also make connections to prior learning. She concurred with Scott's suggestion to postpone the use of the word "triangularization" until the process is complete. This teacher action mirrored that noted by Hornberger (2002), wherein the author considered it a hallmark of good teaching to pay more attention "to the meanings expressed by the children than to the form in which they expressed them" (p. 84).

Bryce's remaining citations under this code mostly lied in his Academic Language Project submissions and focus on providing official (and technical) definitions of terms to his students. For instance, for the term *rational function*, he indicated in his lesson plan that he would introduce it as follows:

Definition (Rational Function). A *rational function* is the quotient of two polynomial functions:

$$f(x) = \frac{p(x)}{q(x)}$$

where $p(x)$ and $q(x)$ are polynomial functions of any degree

Following his introduction of this definition, he does present more instruction around the word *rational*: "Notice that we call this a *rational* function. That does not mean we are only dealing with rational numbers! Instead, think of this as a 'ratio,' which is why we call them 'rational' functions." While Bryce himself seemed to lean towards a more traditional approach to addressing language in his own practice, he noted more progressive means conducted by a veteran teacher. In his subsequent reflection for his Academic Language Project submission, he remarked,

"In this week of lessons, I observed a teacher (different from my CT) introduce new vocabulary to the class regarding linear equations. She did so showing students a

variety of different lines graphically and algebraically and asking students to compare the differences and similarities between the different lines. Instead of giving the definition to the students, she allowed the students to build a definition from their observations. This is much more difficult to do than giving a definition to students, especially since this class was not an honors level class. Despite the difficulty of this teaching method, however, teaching students how to generalize patterns and observations that they make is an important skill, and this lesson was scaffolded well. By the end of the lesson, students learned two new definitions that they had developed on their own, giving their definitions much more understanding and meaning.”

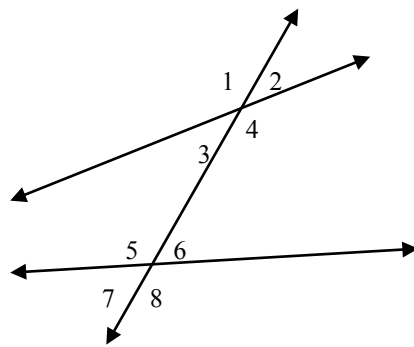
Bryce acknowledged the strength in a more student-centered approach, but applies a level of difficulty to it. Perhaps his reason was because of this method’s unfamiliarity to him and his experience as a mathematics learner, or perhaps Bryce found it “much more difficult” because he had a preconceived notion that more progressive techniques are reserved for “honors level” classes as opposed to being applicable for all students.

Torres-Velasquez and Lobo (2004/5) assert, “Teachers should teach both mathematics and English vocabulary explicitly and in ways that the students will not forget” (p. 253). Mabel’s methodology for teaching vocabulary words seemed to adhere to this methodology and align more to that of the teacher that Bryce observed. She discussed having students make their own meaning and expressing terminology in their own words a number of times.

“In this manner, students were interacting with the graphing process and creating their own visualization of a concept, as well as interpreting a meaning of a new

vocabulary word by examining images that they have created” (Lesson Organizer & Reflection)

“Each graph was displayed and the do now questions were gone over, including the introduction of the new vocabulary term: linear. Students were able to see that linear meant a line, and were also able to visualize this vocabulary term with the linear graph they had just created.” (Lesson Organizer & Reflection)



“c.

- i. Review of interior angles:
 1. Ask what interior means
 2. Ask for examples - angles 3, 4, 5 and 6
- ii. Review of exterior angles:
 1. Ask what exterior means
 2. Ask for examples - angles 1, 2, 7, and 8
- iii. Corresponding angles: angles 1 and 5
 1. Ask for additional example
- iv. Alternate interior angles:
 1. Ask what alternate means, and ask what alternate means when we are talking about interior angles

2. Ask for example: angles 3 and 6 (one example)
- v. Alternate exterior angles:
 1. Ask what alternate means when we are talking about exterior angles
 2. Ask for examples: angles 1 and 8 (one example)
- vi. Same-side interior angles:
 1. Ask what same side means in terms of interior angles
 2. Ask for examples: angles 3 and 5"

(Academic Language Project, Lesson Plan #1)

In the latter example, especially, Mabel's plans indicated her teaching of terms that may be used differently in other contexts. Lucas and her colleagues (2008) noted, "students need to understand the ways language is used in the subjects they study in schools" (p. 365).

Whether the words or phrases are being reviewed or introduced, Mabel used students' backgrounds and prior mathematical knowledge to make connections for themselves to the content and terminology at hand, and, thus, build towards fluency in the mathematics register in a meaningful and deeper way. In one of her reflections for the Academic Language Project, Mabel observed a language issue for students taking a Geometry test that triggered for her the importance of language comprehension in relation to students' ability to demonstrate their conceptual knowledge:

"One problem asked for students to indicate how we would know if a set of lines were perpendicular. Two of the multiple-choice options contained the word 'product.' For many students, this term is introduced in early academic years, but it was clear for some students, that there was a misunderstanding of the meaning.

Product can have a multitude of definitions and meanings. It can mean something that is created or being sold, or, in mathematical terminology, it is the result of multiplication. It is easy to understand why some students, regardless of their achievement level, would be confused by this terminology on a test... Similarly, I noticed that students, who were confused by this terminology, did have a clear grasp of the concept. Therefore, it would seem that a misunderstanding of mathematical vocabulary could hinder mathematical development.”

Following this observation, Mabel affirmed her belief in approaching vocabulary instruction in more connective manner:

“I believe that in my own classroom, I will ensure that I am employing mathematical vocabulary that has been used in previous academic years, as well as terms that are new to students. I think this is essential in supporting students’ advancement in mathematics and important to reduce the issues students have with terms with multiple meanings.”

Use of Everyday Language

The category, *use of everyday language*, was the set of strategies within the comprehensible input component of the conceptual framework that was least used among the five participants: Bryce – 3 total instances; Hank – 0; Mabel – 8; Natalie – 1; and Scott – 1. Comprised of two codes, *everyday language to define math terms* and *everyday words with different meanings in math*, this category offered useful tools for assisting students in making connections from their existing language knowledge to that of the mathematics register. Bryce utilized everyday language and experiences to explain symmetric functions:

“To understand a symmetric function, think if you were making an omelet or a taco. If you drew half of the graph of your equation on one side of the omelet/taco and then folded it over, it would appear the same on the other side, except as a ‘mirror image.’ A quadratic equation is also a mirror image, because one half of the parabola is always the mirror image of the other.” (Academic Language Project, Lesson Plan #2)

In his Lesson Organizer and Reflection, Bryce pointed to the multiple meanings of the word *factor*: “For example, in our discussion of factors, I related factors to their previous understanding of factors of a number, as well as colloquial definitions of the word ‘factor,’ such as, for example, ‘Smoking was a factor in the person's cancer.’” Such instruction was important in assisting Bryce students in “becoming more precise and reflecting more conceptual knowledge” (Moschkovich, 2004, p. 194), by guiding them through a process of refining their mathematical descriptions.

Brisk and her colleagues (2002) concluded, “A student’s stronger language is the best medium for thinking and understanding” (p. 115). For many students, this stronger language was conversational, or “everyday,” language. Moschkovich (2004) noted, “Although differences between the everyday and mathematical registers may sometimes present obstacles for communicating in mathematically precise ways and everyday meanings can sometimes be ambiguous, everyday meaning and metaphors can also be resources for understanding mathematical concepts” (p. 196). Such was the case when Scott used less technical language to build students’ understanding of geometric medians, reporting in his Lesson Organizer and Reflection submission,

“Instead of the academic language used in the textbook (which they had seen the day before), I used simple definitions such as ‘a median cuts one side of a triangle in half,’ emphasizing the word side. This seemed to help the students understand the words better, and they were subsequently able to answer Problem I.”

Mabel submitted the remaining examples of the *use of everyday language* category. For instance, when describing in her Academic Language Project reflection a lesson involving the reflexive property, Mabel noted, “the students defined it as ‘everything is equal to itself,’ which is similar to the textbook definition. This student definition, however, carries more meaning because it came from the class and not the book.” In her Lesson Organizer and Reflection assignment, she remarked on students’ word knowledge from other environments leading to their understanding of independent and dependent variables in mathematics:

“Students found that defining independence was easier because they had heard this word before. Examples were offered, such as, if a student works independently, that means the student is working individually and does not rely on outside assistance. When it came to defining the word dependent, students were asked to think about the word in terms of their definition of independence. Students were then able to define the word as something that is not individual and needs assistance. In terms of the questions about the graph, students were able to talk through what the variables of the graphs were, and which variable was independent, and did not ‘depend’ on another variable. This was aided by the student realization that there was a variable, such as, the total cost, that was dependent on another variable, which was the number of items bought (the independent variable). In this manner, students were

able to connect vocabulary to their prior knowledge, which allowed for them to build on new vocabulary and make connections to the algebraic content of the lesson.”

While the participants did not utilize everyday language for mathematics instruction on a frequent basis, perhaps they avoided relegating language use in their classes to isolated vocabulary, as Moschkovich (2002) purports, “Focusing on the obstacles between the everyday and the mathematics register can obscure how everyday meanings can be resources for mathematical discussions. [This perspective] can be interpreted as reducing mathematical discourse to the use of vocabulary or presenting a deficiency model of bilingual students as mathematics learners” (p. 206).

Direct vs. Indirect Input

In the process of sorting the emergent codes according to the elements of the conceptual framework, a difficulty arose in compartmentalizing the codes for comprehensible input. The issue was that there were some codes that were direct implementations of this element and others were more indirect, as they denoted factors that must be put in place or considered in order to allow the more direct input to occur. Some of these codes pointed to teacher beliefs, such as *constructivism vs. positivism*, *effect of prior experience* [as a learner], *math as [a] language*, *question of equity in math instruction*, and *universal nature of mathematics*. Other codes like *class culture or management* and *concern for task length* involved how a class might operate. Interestingly, there were 17 codes deemed “direct” comprehensible input and 33 indirect. As implications of this study are considered, codes in this secondary layer of comprehensible input may offer another

lens through which the data can be viewed or even another line of research to be considered in the future.

Results for Student Engagement

Lucas and her peers (2008) suggest that some kind of social interaction wherein ELLs are required to use language and negotiate meaning is crucial for developing their fluency in academic language. This necessitated the role of student engagement in the conceptual framework of this study. For the related codes that surfaced in the participants' materials (see Table 6.3), there were just over half as many codes as the "direct" input codes, but just as many categories. These groups were *student actions*, *collaborative interactions*, *teacher assistance*, and *student dispositions*, with the most number of codes falling within the *teacher assistance* category (60) and most even distribution of occurrences among the participants falling within *collaborative interactions* (50). Neither of these groupings, nor the remaining ones, followed the increasing pattern that would have been the ideal outcome of the data. However, looking across assignments, there were larger and similar numbers of citations between the Case Study Analysis and Lesson Organizer and Reflection assignments, as well as notations by almost all of the participants within these two submissions.

Teacher Assistance

The *teacher assistance* category was overwhelmingly dominated by the *questioning techniques* code, which had 55 occurrences representing all five participants. Mabel's material culture exhibited more often than all of her peers in this study (27).

Understanding the importance of how questions are phrased for students, Mabel posited in her Case Study Analysis, "If a teacher fails to word a question effectively, all students will be

disadvantaged, including students who qualify for special education or are English Language Learners, in which academic language is a key component to understanding mathematical content.” In her own lessons, Mabel used questions to guide assist students in clarifying their prior knowledge. In her Lesson Organizer and Reflection, she reports,

“When I did encounter students who were not sure how to proceed with the activity, I asked them to recall graphing functions and how they began with that process. Students thought about how they did this, which was typically done by setting up a table of x values and corresponding y values. I asked students to recall that when we had our x values, we could input them into the function and receive a y value. When students remembered this process, they were able to proceed with the do now activity.”

Mabel also employed questioning in this lesson in order to have student examine and express their own thinking as well as make meaning of a newer concept. This correlates to Lucas and her peers' (2008) suggestion for teachers to “modify they talk to ask *how* and *why* question, as well as questions to which they do not know the answers” (p. 369). An example of Mabel’s implementation of this skill was in her Lesson Organizer and Reflection:

“Students were also asked which graph they believe is linear and why. In this manner, students were interacting with the graphing process and creating their own visualization of a concept, as well as interpreting a meaning of a new vocabulary word by examining images that they have created.”

She continued this type of inquiry in her Academic Language Project submissions:

“a. Students are asked if they agree with the conjecture proposed by their peers.

b. Students are asked how they came to that conclusion.” (Academic Language Project, Lesson #1)

“Ask how they know that the lines are parallel. Can they check by using a protractor? (The lines are parallel because of the Converse of the Corresponding Angles Postulate.)” (Academic Language Project, Lesson #2)

“a. Ask students if they notice a pattern or a relationship between the x-values and the y-values

b. Identify what that pattern is – can this be written as an equation?” (Academic Language Project, Lesson #3)

“a. How are these graphs similar? (Think back to the lessons of the past two weeks – what have we been learning about? Are these graphs functions?)

b. How are these graphs different? (By appearance, how are they different?)

c. What does linear mean? (Pass out Graphic Organizer)

i. How can we tell just by looking at the graph that these graphs are linear?

(One forms a line, one is curved and does not look like a line)” (Academic

Language Project, Lesson #4)

With these lines of questioning, Mabel had the potential of engaging students in conversations wherein “the teacher acts as a facilitator rather than a questioner” (Lucas et al., 2008, p. 369). At the end of Lesson #3 of her Academic Language Project submission, she asked students to respond to three questions as a lesson summary and exiting exercise:

“1. What was one thing you knew before today’s lesson?

2. What was one thing you learned from today’s lesson?

3. What was one thing that is still unclear from today’s lesson?”

Bryce had the second highest number of instances of this code in his artifacts (18), and when asked during his post-semester interview about his frequent mentioning of questioning and questioning techniques in his work, he responded,

“... because mostly my supervisor has started mentioning like I use the same style of questioning a lot, and I noticed that. And then she actually like wrote it down how many times I did this one way that I question students. It was like, she like had to like, I asked them to fill in the blank a certain number of times, and I asked them like for one single word answers, not just like yes or no, but like five or a factor, or something like that. Like, this many times. And she showed how it wasn't very balanced the way that I was using different questioning styles. And I never even realized that... So then, that was something that I'd been fixing, or trying to mix it up a little bit like, to like, sort of make sure I'm getting different kinds of questions at them.”

The other participants also discussed the questioning style of the teacher in the Case Study Analysis. Hank remarked,

“While reading the marble case and the interactions that Mrs. Elmore had with her class I got the sense that some of her questions where shallow and she had something specific in mind that she wanted to hear from the students. She asked questions such as; ‘What does a line always have?’ and ‘If the line has a slope, what is it?’ These where questions where the students needed to respond with a particular answer and this types of questions did not encourage the students to explore and investigate.”

Natalie felt, “Mrs. Elmore was too insecure about her questioning techniques surrounding this experiment.” She continued, “I found that she was unsure as to how she was going to be able to convey each of the separate understandings, or how to address the problems raised by the students.” However both Mabel and Scott were more positive about the teacher’s technique:

“I thought Mrs. Elmore’s questioning techniques were effective for her students. I thought that her questions were often sequential and logical, which are aspects of effective questioning, according to Posamentier, Smith, and Stepelman (2010).”
(Mabel)

“I like Mrs. Elmore’s questioning techniques, and I feel that they would have been very effective if she had had a clear understanding of the content. I think that it is almost always better to get the students to arrive at knowledge via a series of questions, rather than simply telling them everything.” (Scott)

The influence of his cooperating teacher permeated Bryce’s comments regarding the questioning techniques employed in his Lesson Organizer and Reflection:

“The questioning style that I employed focused mainly on the problems, with questions such as, ‘Why did you say that this was a factor of the polynomial?’ or, ‘What theorem can we use to determine the potential factors of this polynomial?’ In retrospect, I would have liked to include more questions that promoted metacognitive awareness, where students began to understand the deeper meanings behind factoring and zeros of a function. For example, a good question to relate factoring to their previous understanding of factor numbers would have been: ‘Why do you think that we I did a polynomial by one of its factors that I have no

remainder? What does this say about the process of factoring?’ Allowing students to see broader connections enlightens their understanding of the conceptual connections within mathematics.”

Mabel also considered questioning in her Lesson Organizer and Reflection, noting how used this method to alleviate confusion and to assist students in accessing their prior knowledge. She reported,

“When I did encounter students who were not sure how to proceed with the activity, I asked them to recall graphing functions and how they began with that process. Students thought about how they did this, which was typically done by setting up a table of x values and corresponding y values. I asked students to recall that when we had our x values, we could input them into the function and receive a y value. When students remembered this process, they were able to proceed with the do now activity.”

According to Hornberger (2002), this ability to assist students in retrieving information they learned previously could be considered a means of “compensat[ing] for her inability to exploit a common reservoir of community-based knowledge by drawing on students’ classroom based knowledge from previous lessons” (p. 85). Scott corroborated this use for questioning in Lesson Organizer and Reflection, commenting, “Rather than simply telling them the definitions, I tried to ask questions and be as much of a facilitator as possible.”

Student Actions

For the portion of codes considered as *student actions*, most of the participants’ emphasis was placed on communication, whether oral or written, and hands-on experiences. Freire (2005) stressed the significance of communication, writing, “only

through communication can human life hold meaning. The teacher's thinking is authenticated only by the authenticity of the student's thinking" (p. 77). In his Case Study Analysis, Hank echoed this notion in his comment about the importance of students' verbal exchange: "Also I believe there is a great benefit to working with others, the ones who need the help gain assistance from the others explaining the material and the ones who are explicating profit from clarifying the concept to others."

An example of the authenticating of a teacher's thinking through that of one's students occurred in Hank's Lesson Organizer and Reflection as the class discussion continued: "When the conversation shifted to talking about what else, besides the slope, was needed to find the equation of a line students were also raising their hands and shared with the class. One of the students shared that to write a linear equation we also need to find the y-intercept of the equation." For Mabel's Lesson Organizer and Reflection submission, her students used written communication as the means of engagement:

"I began this lesson by asking students to demonstrate their prior knowledge by graphing two equations, one which happens to be linear, and one nonlinear.

Students noted the similarities (i.e. that they are both functions) as well as the differences (i.e. one is a straight line and one is curved). Students were also asked which graph they believe is linear and why. In this manner, students were interacting with the graphing process and creating their own visualization of a concept, as well as interpreting a meaning of a new vocabulary word by examining images that they have created."

According to this selection, in this communicative process, not only are students displaying their graphing skills, but also they are exercising process-related skills, or what the

Common Core State Standards (2011) would refer to as “Standards for Mathematical Practice.” Students were required to construct arguments and justify them, applying reasoning skills, as well as compare and contrast graphs, looking for structure, in order to develop their own conjectures – skills crucial in building students’ ability to communicate mathematically (Moschkovich, 2004).

Furthering the notion of student engagement, all of the participants cited aspects of the Case Study Analysis in regards to hands-on activities, as the case was based on a classroom experience wherein the students used a pebbles-in-water experiment to develop a linear equation. The participants remarked on the importance of students having these kinds of learning encounters to provide access to the language and the content, but also to deepen learning.

“In this experiment, the slope (rate of change) was being represented by observing the rate at which the water level changed, as we added (changed) the amount of marbles in the water... This experiment was designed, and meant, to more deeply explore the meaning of slope in a real life situation.” (Natalie)

“By using this type of activity, combined with questioning techniques, Mrs. Elmore also benefits ELLs and students with disabilities. The experiment with the marbles and the rising water level creates a concrete, meaningful context for language such as slope and intercept. Allowing the students to communicate their understanding in their own words is especially beneficial to ELLs, because it gives them practice expressing ideas coherently in English.” (Scott)

“First, mathematics instruction should be a contextual learning experience that combines both theory and application. Mathematical theory forms the foundation of

learning, and provides students with a variety of different tools and skills. However, the learning of theory should be framed in the greater context of application, with students utilizing the skills they have learned in class and actively applying them to patterns and situations that they recognize in their own lives.” (Bryce)

“From her class I believe that students of all types could benefit from the exploratory investigation where they would be working hands on to find and use data to understand the slope of a line. This type of instruction can assist diverse learners.” (Hank)

“The exploration provided a hands on experience of slope and a rate of change that can be beneficial for all students, instead of only learning slope from problems in a book... When students are the one's exploring ideas through discussion, or hands on approaches, they are creating a classroom environment in which the students are also the teachers.” (Mabel)

For each of these instances, all students’ learning would be aided from participation in the class activity. Brisk and her peers (2002) stated, “Teachers should make approaches that are beneficial for bilingual learners their preferred approach to teaching, either with the whole class or selected groups” (p. 113), as “Facilitating instruction for bilingual learners renders highly adaptable teaching that address classes with a variety of skills and backgrounds.” Thus, strategies helpful to linguistically diverse students in Ms. Elmore’s class were likely helpful to all students in her class.

In the Lesson Organizer and Reflection assignment, Natalie was adamant that her students would have the opportunity to explore functions in an interactive manner:

“When planning my lesson on the six (of twelve) basic functions for my honors pre-calculus class, I knew that one of the most important things to me was going to be making sure that the students were interactive and that the material I was using was as interactive and technological as possible.”

She continued, “I found this activity to particularly helpful because it incorporated language skills when the students had to write out the characteristics, application skills when the students had to find or create examples and then solve and plot them on the graph, as well as a good basic review of the material.” For the same assignment, Scott described the purpose of exposing students to a more experimental version of mathematics in terms of Freirian and Vygotskian schools of thought:

“I knew that it was important to have the students try to discover the inequality on their own, rather than just copying it down off the board. For one thing, it decreases the emphasis on the teacher as the exclusive source of knowledge [Freire], and aligns with Vygotsky's theory that students should construct their own knowledge.”

He further maintained, “Learning through activities with manipulatives helps students significantly in constructing this meaning. In my own experience, I have always found that students retain knowledge much better when they are actively involved in learning it.”

Collaborative Interactions

Within the *collaborative interactions* category, the participants discussed the positive and negative aspects of student working together, as well as how they are grouped together in order to perform tasks. Hank commented on his own learning experiences in the Case Study Analysis and the one of the advantages of working with other students: “My personal take on heterogeneous classes is that they represent the best learning

communities for all students. I think about my own learning and school experiences and I have been part of classes where I was able to help others who needed my help and also classes where I benefited from other who understood the material better than I.” While he did not consider this in the design of his lesson for the Lesson Organizer and Reflection, in hindsight, he would have done so: “One of the aspects of the lesson that I would change if or when I teach it again is that I would set up the class in small groups. I think that working in small groups will be a better set up for the students to scaffold one another, check each others' calculators, and help the students stay on task during all parts of the lesson.”

In her Case Study Analysis, Mabel stated her belief in collaborative interaction for mathematics classes: “I believe that cooperative learning is essential in students' exploration, discussion and discovery. In this manner, students will play dual roles as learners and as teachers.” In her lesson, she did not create activities that employed collaboration, but rather allowed students to assist one another as they desired:

“I did notice that as I walked around, those students who were finished with their work offered- to help students who did not understand. I believe that this is beneficial for both students, because it reinforces the knowledge of the explainer, while allowing the learner to learn within their zone of proximal development (Vygotsky, as cited by Albert, 2003). Luckily, this was not an isolated incident. There were multiple students who offered to help their peers. I think the development of my classroom as a safe learning community is beneficial for students. I believe that when students are willing to help one another, there is far more learning occurring than if I was to help every student who may be confused.”

A similar paradox was found between Bryce's reflection in the Case Study Analysis and how he approached his lesson for the Lesson Organizer and Reflection. In the Case Study Analysis he indicated a commitment to allowing students to build their own knowledge through collaboration.

"Both group and individual learning experiences are incorporated into a comprehensive collaborative learning process that includes all students as well as the teacher. In my classroom, I do not view myself as imparting knowledge to the students; instead, I am a guide for the students to discover mathematics that they will constantly build upon as a lifelong learner."

No group work was actually implemented until after this lesson was taught and practiced, and when collaboration was introduced, Bryce was looking to document student learning: "This assessment task was organized into a relay race. The students were split randomly into two teams by their seating arrangement... Although the students were working in groups and could help their teammates, all students were required to show their work at least once to me to receive a 'point' for their team." Perhaps these examples demonstrated a conflict that Bryce was trying to reconcile between the more traditional approach to teaching mathematics and the more progressive model that the course promotes and better embeds LRP. Brisk and her peers (2002) advised,

"Approaches chose to address the needs of bilingual learners will be sustained if they are incorporated in the regular class routine. When the needs of bilingual students are seen as extra, the changes will never take place or may not last because they are perceived to be an added burden" (p. 113).

Additionally, Freire (2005) warned, “Those truly committed to liberation must reject the banking concept in its entirety... They must abandon the educational goal of deposit-making and replace it with the posing of the problems of human beings in their relations to the world” (p. 79).

Natalie expresses her beliefs in collaborative learning in her Case Study Analysis and later attempted a collaborative assignment in her Lesson Organizer and Reflection:

“I believe that students can help each other learn in ways that teachers aim to do themselves. In my classrooms, group work is imperative. Most students have the ability to listen to what is said in lecture, or read a textbook, and then restate that same thing in class to their teacher. But for a student to be able to explain their understanding to another student, or even more so for a student to be able to help another student understand, is in a language that they are not only more comfortable with, but one that will stick with them.” (Case Study Assignment)

It is interesting that she confesses apprehension about releasing her students to work in groups in her Lesson Organizer and Reflection, but it turned out that there was no need for such angst:

“I was nervous that of three things: that the students would just talk amongst their friends instead of focusing, that the students would not have understood the material well enough to complete the activity, or that we would simply run out of time before finishing. My worries were, to my pleasant surprise, all unnecessary. Although there was some chatter through out the classroom, between my circulating the classroom and the natural hustle and bustle from the students, everyone finished their posters.”

Bryce also expressed some hesitation about having students work in groups:

“Although the assessment was reflective of the material, for struggling students, it provided them with only the opportunity to learn from more advanced students.

While this was helpful, it was unclear through the structure of my assessment whether the students were able to build their knowledge. In future lessons, I would like to structure collaborative learning activities so that all students may enhance their learning, instead of struggling students simply relying on advanced students as a crutch.”

Freire (2005) cautioned, “The revolutionary society which practices banking education is either misguided or mistrusting of people. In either event, it is threatened by the specter of reaction” (p. 78-79). Nevertheless, Bryce noted, “The students however, did seem to enjoy the variety of different techniques that I employed as well as the activity to test their learning. Using group assessments along with differentiated instruction will most certainly be a part of my future lessons.” These seemed to be corroborated by his observation, “As the communication within the groups was surprisingly tame, most likely due to the class’s small size, if a student answered a problem incorrectly, I could infer that it may be due to a lack of understanding of at least that individual student.” Natalie’s positive experience with such an instructional approach led her to even conclude, “It was the most successful lesson I feel that I’ve ever taught in terms of student focus, participation, and understanding.”

Results for Comprehensible Output

Comprehensible output, as defined by its originator, is the requirement that students be “pushed toward the delivery of a message that is not only conveyed, but that is conveyed precisely, coherently, and appropriately” (Swain, 1985, p. 249). This practice

serves three primary functions (Swain, 2005): to reveal one's own prowess with the new language to the learner (i.e. *noticing/triggering*), to allow the learner to test one's own ability to convey the meaning in the new language that was intended (i.e. *hypothesis testing*), or to have the learner examine the language he or she produces as it is utilized within a sociocultural context (i.e. *metalinguistic (reflective)*). In this study, comprehensible output was coded according to these three functions, with most of the notations lying within the Case Study Analysis, Lesson Organizer and Reflection, and Academic Language Project submissions, noted once in the Critical Analysis assignment, and absent in the Imagine Ifs (see table 6.4). The lack of occurrences in the latter assignment was more than likely due to the nature of the prompts, which were geared more towards lesson design and student engagement. Also, since there were only three codes applicable to comprehensible output, these were also the categories used for this part of the conceptual framework.

Hypothesis Testing Function

For the notion of hypothesis testing, students are given the opportunity to go beyond using the language and determine whether their intended meanings are relayed. Thus, instances where pupils were allowed to report out to someone were searched for within the participants' materials. This function of comprehensible output bore the most number of occurrences of the three functions, 42 compared to 16 *metalinguistic* citations and 17 for *noticing-triggering*. Most of the hypothesis-testing citations were identified in Mabel's Academic Language Project:

“Ask students what the postulate means (that if there are parallel lines cut by a transversal, the corresponding angles formed are parallel).” (Lesson #1)

“Students work either together or individually on the worksheet to write conjectures in their own words regarding the measures of the remaining angle pairs... Students volunteer to write their conjectures on the board.” (Lesson #1)

“2. Return to the Corresponding Angles Postulate (Ask what it is – write on board)

- a. What is the converse of this postulate? (What is a converse? What do we know about converses typically? This converse is true.)” (Lesson #2)

“a. Is this a linear function? What are the domain and range?

What if the x = the number of people and the y = number of slices of pizza? Can we have negative humans? Or $\frac{1}{2}$ of a human? No – this changes our domain and range” (Lesson #4)

These excerpts are also directly tied to her questioning technique, categorized within the student engagement portion of the framework. Similarly, all four of Bryce’s Academic Language Project quotations for the hypothesis-testing function all fell within Lesson #2 and three of them were linked to his questioning style as well:

“What do important things do you notice about it? Accept 2-3 answers from students. Make notes of their responses on the board.”

“Do you remember what our criteria was for determining a maximum and minimum value? Students should recall the chart that from earlier in the lesson.”

“Does anyone remember different words that we used for the x -intercepts? Solutions, roots, zeroes.”

Bryce’s instances of comprehensible output, in general, were sparse with a total of six for hypothesis testing, three for metalinguistic functioning, and none for the noticing-triggering aspect.

In Scott's Case Study Analysis, he reiterated the importance of students being allowed to communicate for their own benefit, "Allowing the students to communicate their understanding in their own words is especially beneficial to ELLs, because it gives them practice expressing ideas coherently in English." He continued and expresses the teacher's responsibility once the student tests his or her language ability, "Since students are answering in their own words, they may say things that are ambiguous or incorrect, and it is the teacher's job to clear up these misunderstandings immediately." Brisk and her colleagues (2002) corroborated this practice in one of their studies, noting that one of the ways in which to assist students in acquiring language mathematics is to model appropriate language use and explicitly correct student responses wherein grammar, spelling, or punctuation is not used properly.

Natalie specified a way for pupils to operate the hypothesis testing function, "Student presentations are a way to ensure that the students are comfortable with the topic at hand as well as allowing them to get creative as well as work on language and articulation skills." Natalie also mentioned cooperative learning as a method for leading students into this function, "This style of activity allows students to observe the mathematics without language barriers, to describe their observations and results in their own words, and then they can work with the students in their group or Mrs. Elmore to correct any language/vocabulary barriers that they may have."

Mabel noted an instance of hypothesis testing being utilized in her Case Study Analysis, wherein the word choice is not mentioned, but it still demonstrates students being given the opportunity to express themselves for the benefit of their own language and content knowledge development:

“Similarly, when Mrs. Elmore asked what a line always had, slope was not the initial answer of the group, however, the group did seem to associate slope with the data because they had attempted to find slope using the data they collected as their x and y points. Students were also able to identify what tabular values were the values on the x-axis, and which were the values of the y-axis.”

Bryce highlighted a possible missed chance for utilizing the hypothesis testing function,

“During almost the entire time when she [Ms. Elmore] spoke with students about the assignment, she would always ask, ‘What do you think?’ If she was speaking to a group as a whole, she did not correct a student’s answer, nor did she acknowledge if a student’s answer was correct. If a student was incorrect, she merely posed the same question to a different student in the group with little to no response.”

Bryce noted that this excerpt not only indicated a need for variety in questioning, but also, because Ms. Elmore did not respond to the pupils in the case study, the students who were acquiring the academic language had no way to gauge if their word choices contained the desired meanings. Hank continued the criticism of Ms. Elmore’s style, stating,

“I got the sense that some of her questions were shallow and she had something specific in mind that she wanted to hear from the students. She asked questions such as, ‘What does a line always have?’ and ‘If the line has a slope what is it?’ These were questions where the students needed to respond with a particular answer and this types of questions did not encourage the students to explore and investigate.”

Even though the questioning style may be close-ended hindering the exploratory possibilities of the activity and perhaps pushing the exercise close to the noticing-

triggering border, students are being given some space to produce language and gauge listeners' reception of their intended meanings.

In her Lesson Organizer and Reflection, Mabel described an instance wherein students are given the occasion to use language in a hypothesis-testing manner, "Students noted the similarities (i.e. that they are both functions) as well as the differences (i.e. one is a straight line and one is curved). Students were also asked which graph they believe is linear and why." Natalie also tells of the use of hypothesis testing in her practice, "I was able to get many students to participate by just calling on them to answer questions, fill in the blanks, or give me examples to do on the board; many student's were also very comfortable stopping me if they had any concerns or questions about the material." She later shares her desire to have students utilize more language, i.e. experiment more with meaning relay, via whole-class presentations, "If given another chance with this activity, I would like for the groups to present their posters and for each member of the group to present one of each of the sections on the poster." Bryce, however, used this style of student output more as means of teacher assessment, rather than noticing the value for the students to self-assess their ability to convey the meanings they intend:

"As each student was required to submit their work at least once, if a student struggled with a problem, I could infer if it was due to a lack of individual or group understanding through monitoring group interactions... if a student answered a problem incorrectly, I could infer that it may be due to a lack of understanding of at least that individual student."

Metalinguistic Function

Functioning of this nature occurred mostly during the collaborative interactions noted by the participants, but were fairly isolated with five in the Case Study Analysis submission, seven in the Lesson Organizer and Reflections, and four in the Academic Language Projects. Hank's one citation was the same mentioned in the *Student Actions* portion of this chapter, describing the benefit of students explaining their work to one another. Scott furthered this thought, stating, "the students' understanding of the concepts, not to mention their language ability, will be reinforced more strongly if they participate in the dialogue rather than just listening to the teacher." After all, it is through this exchange of ideas that "The teacher is no longer merely the-one-who-teaches, but one who himself is taught... they become jointly responsible for a process in which all grow" (Freire, 2008, p. 80).

Scott later described a dialogue that ensues regarding the experiment that the class in the Case Study Analysis conducts, but that does not quite take advantage of the possibility for metalinguistic functioning:

"It is impossible to determine how well Tom understands the concept, especially since Mrs. Elmore ignores his question about why it does not matter which points you use to find the slope. Becca seems to get it, although it is hard to tell for sure. She seems to understand that Veda's explanation is incorrect, and correctly states that four marbles makes the water go up 1 centimeter. However, Mrs. Elmore had said the exact same thing seconds earlier, so it is hard to tell if Becca really understands the concept or is just repeating the teacher. Mrs. Elmore follows up by

asking a yes/no question, so again it is hard to tell if Becca really understood her statement about slope or was simply saying 'Yeah' without thinking.

Andie made an excellent insight about not connecting her points, but it is hard to tell how much she or Sam understands the concept of slope. Liz, David, and Greta seem to have the best understanding of the concepts, although Greta's statement about 'how much the water goes up when you put the marbles in' is a bit ambiguous. She needs to be careful not to confuse the water level with the change in the water level. Mrs. Elmore does nothing to clear up this ambiguity."

This excerpt pointed to the critical role that the instructor has in designing activities and questions that guide students into dialogue, but also into self-assessment, wherein student begin to operate metacognitively and self-correct not only their computations, but also their language. Natalie extended this thought, considering how students were grouped for this activity, "For example, if Mrs. Elmore had broken up the students so that Tom was working with a higher ability student such as Greta, she may have been able to more fully explain the concepts of the exercise in a way that Tom may have more fully understood." Thus, Greta would have been able to initiate the dialogue required for the metalinguistic function, but this also may have served as a window for Tom to engage more in the group conversation, gleaning more from the peer dialogue as well.

Scott expounded upon the usefulness of heterogeneous grouping in his Lesson Organizer and Reflection, reflecting on the activity he designed for his lesson,

"People who struggle with math would benefit because it makes the Triangle Inequality more concrete, and also because students are paired up and allowed to work at their own pace. In several pairs, a brighter student helped a slower student

understand why it was impossible to make a triangle from a 2 inch straw, a 3 inch straw, and a 5 inch straw.”

Again, whether the cognitive delay is language- or content-based, have a higher functioning student to begin the conversation may be an opening for the lower functioning student to become more involved in the lesson and begin to utilize skills acquired through the interaction with his or her peer. Bryce discussed this same concept, but limits ELLs to “observation[s] of other students” and to “receiving help in completing the problem,” negating the power of the vacillation of ideas between students. It is because synergistic nature of dialogue that Natalie noted, “as I circulated the room I tried to make sure that even the quieter students were giving someone input, and had a chance to participate.”

In Lesson #4 of the Academic Language Project, Mabel even described a possible progression through the three functions of comprehensible output in a portion of the lesson:

“I think if I noticed my students having difficulty noticing the similarity between the problems, I would display problems that were similar and ask them what it means to ‘solve for’ something, and then discuss how they would solve each of the problems, justifying each step as they proceed. I would then ask them to compare a problem like $3x + 1 = 7$ to $Ax + M = k$ and talk with a partner about why they are similar and how they could solve for the variable x . Students could write on a piece of paper with their partner how this would be solved, with justification. In this manner, students can reinforce their knowledge of how to ‘solve for’ variables both orally with a peer and in written form to be passed in.”

Moschkovich (2004) proposed “three perspectives for thinking about how bilingual students learn mathematics: acquiring vocabulary, constructing meaning, and participating in discourse” (p. 191). In the above excerpt, Mabel incorporated these three perspectives in addition to the output functions. She planned to have students determine the meaning of “solving for,” a noticing-triggering function; then moved to having students “discuss” how they would address the given problem, transporting into hypothesis testing. The remainder of the exercise would be centered on student-to-student discussions comparing and contrasting how to solve for “x” in the two equation – metalinguistic functioning.

Bryce’s lone instance in the Academic Language Project occurred when student are working together in the relay race, Lesson #3. The series of questions he asked the teams are discussed among the students prior to responding to the teacher:

“Round 1: Determine if $h(x) = x^2 + 4$ is a factor of $f(x) = 2x^4 - 3x^3 + 9x^2 - 14x + 7$...

Round 2: Determine if $h(x) = 3$ is a factor of $f(x) = 2x^3 - 7x^2 + 4x - 5$...

Round 3: What is the remainder of the following: $(x^4 + 3x + 1)/(x + 6)$? ...

Round 4: Determine if $h(x) = x + 3$ is a factor of $f(x) = x^3 + 4x^2 + 7x - 9$...

Round 5: What is the remainder of: $(x^2 - 2x + 3)/(x - 1)$? ...

Round 6: Completely factor the following into a product of linear factors:

$f(x) = 6x^3 - x^2 - 117x - 140$.”

Again, connected to Bryce’s questioning technique, while these questions are close-ended in nature, they also elicited language comprehension and usage for the students on each team to discuss their calculations with one another and ensure that everyone on the team arrives at the correct answer and can give the correct response.

Noticing-Triggering Function

This function appeared in the material culture of Mabel, Scott, and Natalie only, and what surfaces more so in this function than the others is the expressive communication form of writing. In Mabel's Academic Language Project, she created a worksheet for her students:

"Worksheet: Students exhibit an understanding of the appropriate vocabulary and the corresponding angles postulate by writing conjectures, in their own words, regarding the remaining angle relationships and the significance of their measures."
(Lesson #1)

In her third lesson for the Academic Language Project, she utilized an exit slip and asked student to respond to three questions:

- "1. What was one thing you knew before today's lesson?
2. What was one thing you learned from today's lesson?
3. What was one thing that is still unclear from today's lesson?"

In Lesson #4, Mabel had her students begin the class with opening assignment ("Do Now"):

"Please graph the following two equations:

$$y = 2x - 1$$

$$y = x^2$$

1. How are these two graphs similar?
2. How are these two graphs different?
3. Are these graphs linear?"

She also had her student read aloud and write on the board in some instances, and even observed this form of output in another instructor's classroom:

"Ask for a volunteer to read the postulate aloud." (Lesson #1)

"When students are finished with their proofs, volunteers will be asked to put their proof on the board (one for each theorem)." (Lesson #2)

"Instead of asking students to learn the definitions that are in the book, the teacher asked students to observe problems and define the properties in their own words."

(Reflection, Week 2)

Natalie allowed her students to use their notebooks to perform the poster-creation task in her Lesson Organizer and Reflection, "Students were able to use their notes and their graphing calculators to fill out the poster as completely as possible." As mentioned in the *Student Actions* section of this chapter, Natalie also commented on the value in students having to use their language knowledge to produce these posters, which also assisted in content reinforcement. Scott commented on the questions that Ms. Elmore utilized in the Case Study Analysis, stating, "Questions such as the ones Mrs. Elmore used give the students a chance to reason through the concepts in their minds and put the content into their own words." This is at the heart of the noticing-triggering function. In his Lesson Organizer and Reflection, Scott had his students complete an activity with straws in order to develop the Triangle Inequality Theorem. He reflected,

"Rather than having to comprehend a theorem that is written on the board, they can try out multiple examples and come up with the rule in their own words. Even if they can't write it down or express it in English, they are likely to have a much

better idea of the Triangle Inequality than if they had just copied it down from the board and watched me do a couple examples.”

He also used the end of the class period to have students respond in writing to a summarizing prompt: “Almost all the students did a good job answering the summary question, which I used to assess their language and communication ability.”

Results from the Concept Presentations

The Concept Presentation was the culminating assignment for the secondary mathematics methods course. For this assignment, preservice teachers worked in groups to “select a middle school or secondary mathematics topic, idea, concept, or pedagogical technique and present it to the class” (course syllabus). In addition, each group was required to submit “a one page word-processed statement that outlines the purpose, and a summary of the content of [the] presentation.” Four groups presented during the semester; however, only three of the groups consisted of case-study participants. These groups’ slides and handouts can be seen in Appendix I. The researcher’s role was that of a participant observer, wherein notes were recorded and comments were given orally about each presentation regarding their strengths and weaknesses in addressing language. It is these observations that will be discussed in this section of the chapter.

Hank, Mabel, and Scott’s Group (Group #2)

This group comprised of all graduate students and had one additional member, Joy, who was an ELL from Korea. The team decided to build their presentation around Scott’s Lesson Organizer and Reflection lesson regarding the Triangle Inequality Theorem. Considering the expanded LRP framework of this study, there were many instances of comprehensible input, student engagement, and comprehensible output that were either

employed or overlooked. The group began by reviewing their lesson objectives, which did not include language objectives. (It should be noted that the first group did not incorporate language in their objectives either.) Figure 6.1 shows the opening slides, including the “Do Now,” or warm-up activity, that the class was asked to do. In the discussion of this task, some terms such as *angle bisector* and *altitude* were addressed, highlighting the vocabulary focus of the team, and yet other such as *median*, *midpoint*, *perpendicular bisector*, and *corresponding* were neglected. Thus, language instruction and accommodations were both focused upon and overlooked within the same task, but in soliciting responses from the class, the group utilized various questioning techniques and the hypothesis testing function of comprehensible output.

In the next part of the lesson, the class was asked to discuss the possibility of proposed scenarios in pairs (i.e. “What Do You Think?”), employing more questioning and initiating the metalinguistic function of comprehensible output. The first prompt inquired, “Can you make a triangle with sides 1 inch, 1 inch, and 1000 feet?” In the whole-group discussion of the task, one of the preservice teachers commented that this case would not produce a triangle because the “arms are too stubby,” which incorporated the use of everyday language into the lesson and allowed for more hypothesis testing of language. Both of these would later serve as the foundation for the Triangle Inequality Theorem. The second prompt, “Can you make a triangle where all three sides are 1 inch?”, elicited the term *equilateral* from the class and more language instruction from the presenting group.

Figure 6.1. *Hank, Mabel, and Scott's Group Opening Slides*

The Triangle Inequality

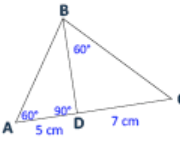


Objectives:

1. SWBAT describe the relationship that exists between the three side lengths in a triangle.
2. SWBAT determine whether it is possible to make a triangle given three side lengths.
3. SWBAT find the maximum and minimum allowable values for the third side of a triangle when given the other two sides.

Agenda:

1. Do Now
2. Pair Activity: The Triangle Inequality
3. Classwork

Do Now

1. True or false:
 - a. BD is an angle bisector of $\triangle ABC$.
 - b. BD is a median of AC.
 - c. BD is a perpendicular bisector of AC.
 - d. BD is an altitude of $\triangle ABC$.
2. True or false:
 - a. $NP = OP$.
 - b. $RS > RT$.
 - c. $NO = ST$.
 - d. $OP > ST$.

The heart of the lesson, the “Pair Activity,” had the class members experiment with straws of various lengths in order to lead the “students” into self-generating the Triangle Inequality Theorem. This was an excellent choice for a hands-on activity that definitely attended to differentiation considerations, scaffolded the content, elicited student and collaborative actions, and enabled students to use the metalinguistic function again. In the culminating discussion of the task, the term *sum* arose; however, no attention was given to its homophonic relationship to *some*, nor its closeness in spelling to *sun*. This was an unfortunate omission of language instruction. In addition, the theorem that was projected to the class, “In a triangle, the longest side must be LESS THAN (and not equal to) the sum of the other two sides” was slightly incorrect. Rather, the Triangle Inequality Theorem states that any side of a triangle must be less than the sum of the other two sides; or conversely, the sum of any two sides of a triangle must be more than the length of the third side.

The classwork portion of the lesson contained a real-world example, to the team’s credit, but was riddled with terminology that may have been completely foreign to ELLs. Words such as *indicated*, *explain*, *between*, *soccer ball*, *straight ahead*, and *mph* are not

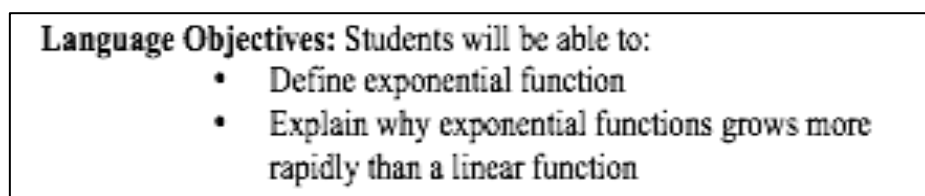
content-specific vocabulary, but rather, are words “that are of high-frequency for mature language users and are found across a variety of domains” (Beck et al., 2002, p. 8). Thus, these Tier-two terms have the potential to stifle students’ understanding of the problem if not addressed in some way. In the summary, the first question asked students to “Find the *range* of possible values for the length of the third side...” However, there was not mention of the word *range* throughout the lesson. In that this word, in particular, has several definitions even within mathematics, the team needed to make clear the meaning of the term in this scenario. Nevertheless, this closing task asked students to write sentences to explain their thinking for both prompts, which allotted for the noticing-triggering use of language in the lesson.

Natalie’s Group (Group #3)

Natalie was part of a team of four preservice teachers. Of these three male students, two were graduate students and one was a fellow undergraduate student. This group chose to focus on exponential functions for their presentation and use the SMART Board. (See Appendix I for their materials.) The “Do Now” asked the course participants to write eight numbers according to their prime factorization, and then simplify the expressions exponentially. The group did not define or discuss the meaning of the terms in the directions – *product*, *prime*, *factors*, *simplify*, or *exponents* – and so the students had to glean from their prior knowledge and possibly tap into their noticing-triggering abilities with language. Not reviewing this vocabulary was acceptable for college-level students; however, some caution was given to the team in considering this lesson for grade-school students who may or may not be ELLs. As the class discussed the results of the task, hypothesis testing of language was activated.

The group then described the goals of the lesson through the use of a lesson organizer, which might serve as a differentiation consideration because of its visual nature. However, it was full of Tier-two and Tier-three language (Beck et al., 2002) that may be unfamiliar to newcomers, and so some thought would need to be given to how it is worded and how to present it – e.g. filled in all at once, have the students fill it in as the concepts are taught, etc. (Beck et al., 2002). This team did include language objectives for the lesson, as shown in Figure 6.2. However, they were inconclusive as to their implementation, as there was no mention of the communicative mode through which definition and explanation were to be given and no indication of collaborative interactions required to supply this information (i.e. in pairs, in small groups, or individually).

Figure 6.2. *Language Objectives for Natalie's Group*



The class was then directed to use a real-world manipulative, M&Ms, to consider to options for increasing the amount of candy one could have. This task was for small groups of students, integrating collaborative interactions and metalinguistic functioning, but there were several concerns. Terms such as *offering*, *doubling*, *option*, and *tally* may be unfamiliar to ELLs and require some language instruction. In addition, the idiom “take a couple of minutes” may necessitate some explanation, as it cannot be interpreted literally. When it seemed most groups had completed the calculation for the task, Natalie and her teammates asked the group members to come to the board and mark their tallies, keeping the lesson

interactive. However, people started using check marks and so this could have led to confusion for younger students.

The next part of the activity required students to put their calculations and expression for Option 2 into a table that was provided and labeled, an excellent content scaffold. Some modifications to the directions so that it was clear what steps to take would have been helpful for ELLs (e.g. step 1, step 2, etc.). Another idiom that may require some clarification was used as well – “figure out.” The final prompt on this page asks, “Can you figure out the equation of the relationship between the number of M&Ms received and the day?” There are two prepositional phrases embedded into a larger one, necessitating either the rewording of the prompt or some language instruction as to how to decode such a question. In addition, “equation of the relationship” should be worded as “equation for the relationship.”

Grids were provided to students for the purpose of scaffolding their content knowledge. The first one asked for the graph of the first five days, and the second allowed for graphing the first ten days. The placement of the graphs’ labels, both the axes and the scales, were a bit awkward, and so it might have been more appropriate to either provide the grids fully labeled or empty, with the intention of having the class or small groups add the labeling together. A class peer or two were selected to come to the SMART board to graph their findings and explain their work, incorporating hypothesis testing again. Natalie and her team provided “challenge” problems to continue students’ application of the exponential function concept. The first question stated, “How would the equation be different if I tripled the number of M&Ms I have you each day?” The group may want to

confirm students' knowledge of the word *tripled*. Beyond this, these questions would attend to students' noticing-triggering abilities with language.

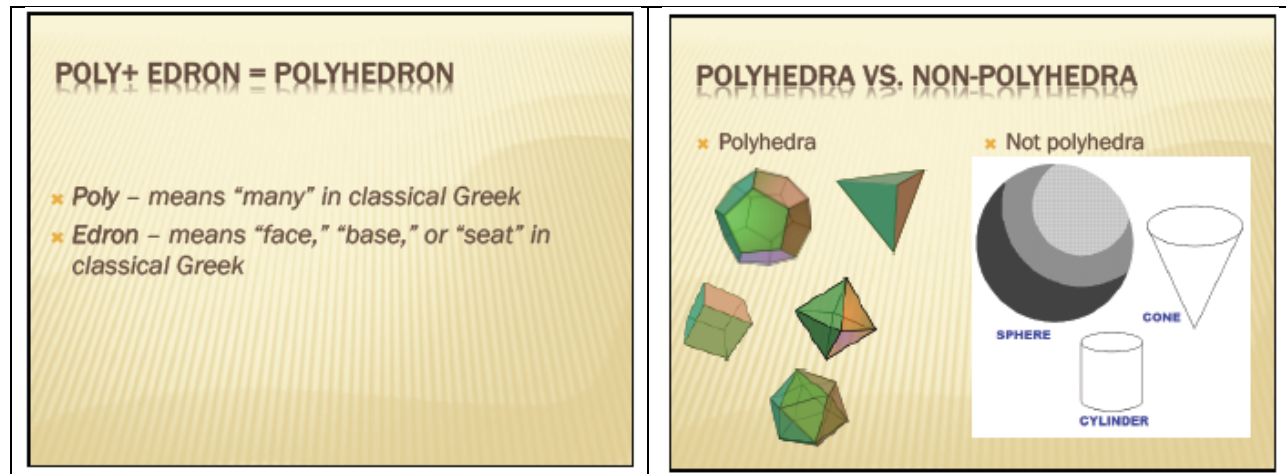
Bryce's Group (Group #4)

Bryce was grouped with two other female undergraduates for the Concept Presentation. This team decided upon having students explore complex polyhedral for their lesson, situating it as the second or third lesson in a three-dimensional figures unit. The group actually began their lesson with a visual of several figures, for which the class was asked, "How would you organize these solids?" This allowed for the noticing-triggering function for the class members, and then when a couple of people were asked to come to the SMART board and sort the figures, a transition was made to the hypothesis testing function. There was a place for some language instruction in this task, as ELLs may not know the meaning of *organize* and the term *solid* is more often used as an adjective rather than a noun.

The next portion of the Bryce's group lesson was a teaching section about polyhedral. Each term was defined utilizing mathematics terminology, with some ties to prior knowledge such as polyhedron being "a solid that is bounded by polygons" – *polygons* being from previous lessons. Figure 6.3 shows how the group deepened the language instruction to include etymology examples versus counterexamples. On other slides, the team also showed not only the Greek prefixes for terms such *dodecahedron*, but also cognates of some of the prefixes in Spanish, Italian, and even German. For *octahedron*, these terms were *ocho*, *otto*, and *acht*, respectively – all of which mean *eight*. Concerns arose because of the amount of vocabulary disseminated to the class without any student action or interaction – thirteen slides worth of terminology, most of which were multisyllabic.

Thus, differentiation of the instruction needed to be considered. Also, perhaps real-world connections needed to be employed in order for students to understand the practicality of the lesson.

Figure 6.3. *Examples of Language Instruction by Bryce's Group*



Slide #14 introduced an activity meant to lead to Euler’s characteristic. (See Appendix I for the accompanying handout.) For this task, the class was provided pictures of nets of three solids, which were referred to during the activity as “shapes” instead, and required to find the number of vertices, faces, and edges in order to calculate Euler’s characteristic. The nets were introduced as “2-dimensional layouts,” which was not incorrect, but there should have been instruction that moved students to the more widely used terminology. There was some informal collaborative interaction between classmates to complete the table and check answers against one another, leading to some metalinguistic functioning. There also was a superficial level of hypothesis testing as the instructors called of member of the class to provide answers for the class display.

The final slide of the lesson led to some whole-class discussion, i.e. hypothesis testing output, as the class had to determine whether or not the net displayed produced a

regular polyhedron, calculate Euler's characteristic, and make conclusions about the results of not having a regular polyhedron. In addition this activity, also instigated the noticing-triggering function. If the net had been distributed to students for actual construction of the polyhedron, the task would have also integrated student actions.

Interpretive Summary

Denzin and Lincoln (2008) describe qualitative research as process like quilt making or creating a montage. "The quilter stitches, edits, and puts slices of reality together. This process creates and brings psychological and emotional unity – a pattern – to an interpretive experience... the use of multiple methods, or triangulation, reflects an attempt to secure an in-depth understanding of the phenomenon in question" (p. 7). This was the endeavor of the multi-sourced approach of this study in order to respond to the research question, "How does a content-and-language integration intervention effect the preparation of secondary mathematics preservice teachers in order to improve instruction for diverse learners, more specifically ELLs, in mainstream mathematics classes?" In addition, this analysis sought to answer the second sub-question: Do the preservice teachers' ability to recognize academic-language challenges for ELLs evolve during the secondary mathematics methods course? If so, how?

The conceptual framework of an expanded version of LRP – namely, comprehensible input, student engagement, and comprehensible output – was the lens through which these questions were approached. The Case Study Analysis, Lesson Organizer and Reflection, and Academic Language Project seemed to have given course participants most opportunity to consider all points of framework. This may be due to the alterations made to the Case Study Analysis and Lesson Organizer and Reflection

requirements by the course instructor for the intervention and the nature of the Academic Language Project, in general. While the Case Study Analysis had previously asked preservice teachers to consider questioning techniques and exploratory activities, the instructor added the prompt:

“How do Mrs. Elmore’s teaching strategies assist ELL and special education students with mastery of the problem? Consider the following strategies:

- 1) embedding language within a meaningful context
- 2) modifying language presented to non-native peers
- 3) judiciously using paraphrases and repetition, and
- 4) consistently negotiating meaning”

(course syllabus)

In the Lesson Organizer and Reflection, regard for ELLs was added to several parts of the rubric by the instructor:

“Includes explicit teaching of the academic language of mathematics (beyond just Tier 3 vocabulary) for all students including English language learners at different proficiency levels” (Section: Completeness of lesson plan)

“How does the use of manipulatives or technology shelter the instruction for English language learners (i.e. make the content concepts comprehensible while providing scaffolded access to language)? Why does this matter?” (Section: Thoughtful reflection on purpose of manipulatives or technology)

“To what extent were efforts made to work within the ELL students’ proficiency levels to make the content comprehensible and to extend their language development?” (Section: Meaningful thoughts about student roles and learning)

“How did you provide opportunities for ELLs to show their understanding of the concepts that were not compromised by their lack of proficiency in English?”

(Section: Insightful thoughts on how the task assesses student learning of the content)

In addition, examples of the lesson organizer graphic with the language objective and other linguistic consideration were provided to students. (See Appendix I.) These modifications to the Case Study Analysis and Lesson Organizer and Reflection offered analysis and planning cues to the preservice teachers that fell within the realm of comprehensible input mostly, which may account for the vast number of citations within this part of the framework compare to the those of the student engagement and comprehensible output portions. Furthermore, with the addition of several readings to the course, as well as the conversion of the core textbook of the course to one that highlighted the needs of ELLs and offered strategies for scaffolding students’ content and language learning, the intervention laid, or at least added to, a foundation for the preservice teachers to practice this learning in their lessons during the semester and in future practice (Lucas et al., 2008).

Based on the course instruction and the participant philosophies expressed in Chapter Five, the number of collaborative interactions and activities planned (or rather, lack thereof) was striking. Rather, the participants generally concentrated their efforts on content-specific vocabulary and questioning techniques and on their own role more so than the role of the learners in their classes. Fillmore and Snow (2002) note, “The competencies required by the various state certification standards add up to a very long list indeed. Perhaps because this list is so long, teacher preparation programs often do not make time for substantial attention to crucial matters, choosing a checklist approach to addressing the

various required competencies" (p. 7). In the case of these participants and this course, the lone methods class for secondary and middle school preservice teachers, I would argue that since there was so much about teaching secondary-level mathematics that had to be taught during the semester, perhaps the participants' foci on their roles as instructors was near their maximum capacity at the time.

To complete the response to how the content-and-language integrated intervention affected the preparation of secondary mathematics preservice teachers, the more prominent codes should be considered. Using a benchmark of 40, the codes with high numbers of occurrences for each framework component were:

- Comprehensible input – *possible language accommodations* (59), *vocabulary focus* (63)
- Student engagement – *cooperative learning* (40), *questioning technique* (55)
- Comprehensible out – *hypothesis testing function* (42)

Every part of the expanded LRP outline, taught during the intervention, was attempted by the participants in order to improve instruction for ELLs and other diverse learners. Plus, based on Mabel and Bryce's Academic Language Projects, it was clear that had all of the preservice teachers submitted the project, there would have been even higher numbers of instances of LRP evident in the material culture. In addition, the Concept Presentations demonstrated that the participants and their peers were giving thought to the language demands of their future students by their use of graphic organizers, incorporation of collaborative activities, and consideration of the structure and etymology of the terminology in their lessons.

The second sub-question of the research question inquired, “Do the preservice teachers’ ability to recognize academic-language challenges for ELLs evolve during the secondary mathematics methods course? If so, how?” There was some evidence of this evolution, considering that strategies for assisting ELLs would not have been incorporated into the participants’ products had not the possible obstacles for these students not been acknowledged. The language instruction and accommodations category for comprehensible output showed a gradual increase in the number of occurrences over the course of the semester. Mabel and Bryce demonstrated this kind of development in several categories due to their Academic Language Project submissions. Use of everyday language, making connections, teacher assistance, and hypothesis testing functions exhibited progression in Mabel and Bryce’s overall ability to recognize linguistically diverse students’ needs and to address them.

Chapter 7

SUMMARY, CONCLUSIONS, AND IMPLICATIONS

Moschkovich (2002) purported, “If mathematics reforms are to include language-minority students, research needs to address the relation between language and mathematics learning from a perspective that combines current perspectives of mathematics learning with current perspectives of language, bilingualism, and classroom discourse” (p. 189). This was the purpose of this study, tying into this line of research through the vehicle of studying the effectiveness of a content-and-language intervention in preparing preservice secondary mathematics teachers to address the linguistic needs of ELLs. Chapter Four provided a quantitative perspective of the participants from the fall 2009 and 2010 semesters in respect to the changes in their beliefs about the nature of teaching and about teaching linguistically diverse students. Chapter Five presented portraits of the five case-study preservice teachers from the 2010 cohort, including their pedagogical philosophies and “ideal classroom” illustrations. Finally, Chapter Six examined the material culture of the case-study participants, with the goal of recognizing patterns and themes across their documentation and Concept Presentations. This chapter will summarize the study, discuss both the quantitative and qualitative findings, offer conclusions and implications, and make recommendations for future research.

Summary of the Study

The intention of this mixed-methods study, which was grounded in the research tradition and methodology of case studies, was to apply the convergence model of

triangulation design in order to determine the effects of the content and language-integrated learning intervention (Creswell & Plano-Clark, 2007). The quantitative arm of the research consisted of pre- and post-intervention surveys from both 2009 and 2010, due to the small number of enrollees in the secondary mathematics methods course each year ($n = 16$ and $n = 14$, respectively). The surveys utilized four-point Likert scales to measure the participants' responses. The qualitative aspect of the study analyzed pre- and post-semester interviews of the five case-study participants in 2010, as well as the drawing prompts from the surveys, the course assignments, the academic language project submissions for the university's Practicum Office, and observation and materials from the Concept Presentations. Tabulation of both the quantitative and qualitative data served as the primary vehicle through which the information was presented, making "the processes employed in the research... more public" (Anfara et al., 2002, p. 35). Through the examinations of this data, the following research questions, as well the sub-questions, were answered:

How does a content-and-language integration intervention effect the preparation of secondary mathematics preservice teachers in order to improve instruction for diverse learners, more specifically ELLs, in mainstream mathematics classes?

- a. Do the preservice teachers' beliefs change regarding their ability to provide content and language-integrated learning opportunities for ELLs in mathematics? If so, how?
- b. Do the preservice teachers' ability to recognize academic-language challenges for ELLs evolve during the secondary mathematics methods course? If so, how?

The conceptual framework for the study was grounded in the literature regarding linguistically responsive pedagogy (Lucas et al., 2008), and then expanded by Swain's (1985, 2005) output hypothesis. The extension consisted of three major components – comprehensible input (Krashen, 1982), student engagement (Lucas et al., 2008), and comprehensible output (Swain, 1985). This approach to the research was undergirded by four assumptions about mathematics:

- 1) Mathematics is more than numeracy and computations.
- 2) Each content area has its own language that goes beyond content-specific terminology (i.e. Tier-three vocabulary).
- 3) Language is a gatekeeper for access to higher-level coursework in mathematics.
- 4) Mathematics teachers are also language teachers.

Theoretically, the study was informed by literature on teaching ELLs and social justice. More specifically, the research on teaching ELLs incorporated notions of how ELLs acquire language (Celedón-Pattichis, 2004; Clarkson, 1992), dilemmas teacher experience while instructing ELLs (Barwell, 2009), and literacy-based strategies for teaching mathematics to linguistically diverse students (Chapin et al., 2003; Lucas et al., 2008; Moschkovich, 1999a). The social justice wing of the theoretical framework considered the experiences of diverse students in American schools historically (Kliebard, 2004), currently (Cummins, 1997; Delpit, 2006; Glazer, 1997; Irvine, 1991), and within mathematics (Cahmann & Remillard, 2002; Gebhard et al., 2004; Gutiérrez, 2002).

Importance of the Study

Lather (1986) advises, “doing empirical research offers a powerful opportunity for praxis to the extent that the research process enables people to change by encouraging self-

reflection and a deeper understanding of their particular situation” (p. 263). This dissertation offered such a vehicle for change by gathering empirical evidence of the effect of a methods course on the preparation of mathematics teachers to support ELLs in learning the concepts and academic language of mathematics. In light of the Common Core State Standards’ (2010) mandate of mathematical discourse and academic language for all students, regardless of language background, in addition to other reform initiatives (Beykont, 2002), new teachers must be introduced to discourse and language, both the theory and strategies. Because the preservice teachers did not all share common language backgrounds nor culture with their students, Hornberger (2002) advises, “Such a teaching challenge requires not one uniform solution but a repertoire of possibilities and alternatives” (p. 85). The intervention, which was administered throughout the course, offered “a broad range of knowledge and skills” (Lucas et al., 2008, p. 362) as research suggests. Participants were instructed in research-based methods conducive to creating learning environments that support ELLs both content- and language-wise.

Survey data was used to examine not only participants’ attitudes about and perceptions of their preparedness to teach ELLs, but also the role of the mathematics instructor in the language-acquisition process; their opinions about ELL inclusion in mainstream classes; their knowledge about modifications and accommodations for ELLs; and their philosophies regarding the teaching, learning, usefulness, and nature of mathematics. In addition, the survey also documented the preservice teachers’ experiences as mathematics learners, as well as their desired future classrooms. Interviews were used to gain more insight into each of the case-study participants’ personal histories, sense of preparedness, responses in course assignments, and survey results. There has been little

empirical research concentrating on the methods and strategies required for the successful instruction of ELLs (Lucas & Grinberg, 2008), and an even scarcer amount that gear these methods towards secondary mathematics. Also, few studies in existence have featured a content-and-language integrated intervention with an entire class of content-specific prospective teachers of the existing literature. Rather, most studies have focused case studies of no more than a few participants. With the triangulation of these various sources of data, a more holistic understanding of the effects of the content-and-language integrated intervention on the preparation of preservice secondary mathematics teachers at this university will be attained, and, thereby, serve as a potential data-supported example for teacher-preparatory institutions considering similar models (Anfara et al., 2002).

Discussion of the Findings

The findings of this study were presented in Chapters Four, Five, and Six. Chapter Four focused on documenting changes in the teacher's beliefs and attitudes for the two semesters of the implementation of the intervention. Chapter Five proffered descriptions of each of the 2010 case-study participants – their backgrounds and their pedagogical philosophies – in their own words and illustrations. This chapter also employed each participant's responses to the survey questions studied in Chapter Four for the validation of their viewpoints via multiple sources of data. Chapter Six rendered a cross-case analysis of these preservice teachers' experiences through the lens of the conceptual framework. The findings from these chapters indicated a positive effect on the beliefs and practices of the participants who underwent the content-and-language integrated intervention in this university course.

Beliefs and Their Links

Teacher attitudes and beliefs about language and linguistically diverse students have an impact on not only teacher-student interactions, but also instructional practices (Bartolomé, 2002; Lucas & Grinberg, 2008). The goal of using the pre- and post-intervention was to capture the change in perceptions of the participants in five thematic areas: preparedness, knowledge, attitudes, role of the teacher, and philosophy about mathematics. The descriptive statistics and paired t-tests produced statistically significant changes in the desired direction for several of the variables. Of the nine questions designed to measure the participants' preparedness to provide language-integrated mathematics instruction, notable improvement was visible in five. Three of the four questions that addressed the preservice teachers' knowledge about teaching ELLs demonstrated significant progress. For example, Question #17 stated, "Teachers should not modify their instruction for the ELL students enrolled in mainstream mathematics classes." The mean response for the pre-survey was 2.04, and the mean of the post-survey was 1.64, resulting in a t-value of 3.667 ($p < .05$). All of the inquiries about the participants' beliefs about their role as more than simply a content teacher, but also a language teacher, indicated positive and marked changes.

The correlation findings for both cohorts combined indicated a significant relationship between the participants' beliefs that content teachers should include language development in their instruction and their feeling adequately trained to instruct ELLs in their initial practice. There was also a significant relationship found between how the participants perceived their training and the preservice teachers' intent to include language objectives in their planning and instruction, which resulted in a significant

correlation to participants' belief in the allowance of students' native languages in class and with teachers' views on modifying their instruction for the benefit of ELLs. All of these findings, as well as the others described in Chapter Four, indicate that the influence of content-and-language integrated intervention not only caused significant shifts in the beliefs of the preservice teachers, but that their change in beliefs in one area seemed to have transferred across almost all of the thematic domains.

The Participants through Their Voices

Guba and Lincoln (2008) promoted voice as a means of enriching qualitative data and increasing the validity of a study. Chapter Five utilized the survey drawings, interviews, and pedagogical philosophies written by the case-study participants themselves in the Case Study Analysis assignment provide mini-biographies for the preservice teachers. Irvine (1991) and Fillmore and Snow's (2002) cautioned about the possibility of the lack of cultural synchronization when teachers cannot relate to the cultural experiences and backgrounds of their students. However, two of the participants were ELLs themselves, two of them experienced being language learners in other countries, and three of them came to the class and the study having had positive and successful teaching or tutorial experiences with linguistically diverse students.

Research has pointed to the revelatory nature of drawings that cannot always be expressed in words (Albert, in press; Haney et al., 2004). The participants' illustrations were nothing short of this. Technology in the form of computer carts or tables were featured in all of the drawings. Both Bryce and Mabel drew Socratic arrangements for their desks; however, Mabel annotated her picture with a description of student-centered, collaborative instruction, compare to Bryce's desire to cling to the traditional notion of

teacher-centered instruction. Hank drew an ideal classroom with his pupils arranged in small groups and the teacher constantly in motion as the facilitator. The only concern was that the group tables featured books and calculators – no manipulatives, no supplemental materials – limiting the kind of tasks that could be assigned to groups and possibly the level and quality of interaction among the students (Lucas et al., 2008). A similar concern arose for Natalie's paired students, with no visible materials, facing the front of the room where a teacher and a student stood discussing a problem.

Chapter Six presented a montage (Denzin & Lincoln, 2008), comprised of the participants' assignments and other documentation in their own words in order to gather more evidence as to the effectiveness of the content and language-integration intervention. The change of the course text and the exchange of several readings to ones that discussed more linguistically responsive pedagogy-related topics offered a foundation for the preservice teachers' lessons and future practice (Lucas et al., 2008). The Case Study Analysis, Lesson Organizer and Reflection, and Academic Language Project seemed to have given course participants most opportunity to consider all points of the expanded linguistically responsive pedagogy (LRP) framework, most likely due to the assignments' alterations for the cause of the intervention as well as the purpose of the Academic Language Project's creation.

Participants' instructional foci generally centered around their own input and on content-specific vocabulary and questioning technique, which offer varying degrees of impact on students' learning depending on the teacher's lesson design. Nevertheless, these two components, in addition to potential language accommodations, cooperative learning, and hypothesis testing, were all high-incident topics in the participants' material culture. In

addition, the participants incorporated graphic organizers, designed collaborative activities, and taught structure and etymology of the terminology in their Concept Presentations lessons. Therefore, it can be inferred that, indeed, the content-and-language integrated intervention affected the preparation of secondary mathematics preservice teachers, especially considering that every part of the expanded LRP framework was ventured.

Logic dictates that if teachers were able recognize challenges for their students, they would also attempt strategies that would assist in alleviating these issues. In the case of this study, the sub-question of whether or not preservice teachers' ability to recognize academic-language challenges for ELLs evolved during the methods course can be answered affirmatively because of the cited strategies from the participants' products. The *language instruction and accommodations* category for comprehensible output showed increases in the number of occurrences over the course of the semester. Instances of *use of everyday language, making connections, teacher assistance, and hypothesis testing function* grew in number over the course of the semester for Mabel and Bryce, in particular, indicating the evolution of their ability to recognize challenges of ELLs and implement strategies to address them.

Conclusion and Implications

Brisk and her peers (2002) conclude, "Instruction that facilitates learning for bilingual students benefits all because it addresses individual differences, enriches everybody's knowledge with experiences of many cultural backgrounds, and creates an accepting classroom atmosphere" (p. 119). However, Fillmore and Snow (2002) argue that teachers lack language-instruction strategies for their content area simply because they

have not been equipped adequately through their educational and professional experiences. It is this gap for which this study was developed. The purpose of this dissertation has been to record the effects of a content-and-language intervention, woven into an existing secondary mathematics methods course, on the instruction of preservice teachers enrolled in the course. The intent was to see aspects of the intervention highlighted in the assignments and other submissions produced by the participants, and in addition, observe a change in their beliefs and attitudes about the teaching of language in mathematics. The data gathered through these multiple means was analyzed and discussed in Chapters Four, Five, and Six, as well as summarized the previous section of this chapter. Thus, this section will reiterate these findings and will proffer implications for other teacher-education programs.

Need for More Practice & Restructuring

While data gathered throughout this study from the preservice teachers indicated the development of their practices towards LRP, the participants also expressed in their post-semester surveys and interviews the need for more instruction about academic language and attending to the needs of ELLs in the mathematics classroom, as well as more practice. The preservice teachers were appreciative for the embedding of the content-and-language integrated intervention into the methods course, as it situated the learning and provided meaningful applications of the language-based strategies within such a technical subject. However, much of the evidence uncovered in the participants' material culture pointed to a superficial grasp or application, at best, rather than an internalization of the LRP-related methods taught in the course. For instance, the code *language objectives* had only eleven citations across the five case-study participants' documentation, plus only one

group included language objectives in the Concept Presentation, in spite of the time spent during one of the lectures teaching the class about these specialized goals and practicing them with the class during the same class. This particular opportunity was one of fourteen class meetings in which everything from NCTM's Principles and Standards (2000) to assessment to Geometer's Sketchpad has to be taught.

Language instruction is a newer aspect of teaching in mathematics, and relatively foreign in the secondary mainstream arena. However, as Gebhard and her colleagues (2002) assert, "all teachers, not just ESL and bilingual specialists, must be prepared to teach students of all backgrounds, including the growing number of language minority students" (p. 220). Therefore, time is needed in order to teach this "fusion" of areas and just as much, if not more, time is required for preservice teachers to practice it so that they can move beyond vocabulary instruction, and towards teaching the mathematics register and Discourse. Some participants in the study proposed the creation of a course dedicated to mathematics and language alone. Fillmore and Snow (2002) even suggest the inclusion of seven different classes for preservice teacher preparation: *Language and Linguistics*, *Language and Cultural Diversity*, *Sociolinguistics for Educators in a Linguistically Diverse Society*, *Language Development*, *Second Language Learning and Teaching*, *The Language of Academic Discourse*, and *Text Analysis and Language Understanding in Educational Settings*.

Other researchers argue that this kind of overhaul currently is not realistic (Feldman, 2002; Richardson, 2002). Additionally, due to the small number of secondary mathematics preservice teachers who enroll annually at this specific university, neither groups' recommendations are likely to occur. Rather, the structure of existing courses can potentially provide the most immediate and most feasible means for providing future

undergraduates and graduate students with the increased language training and rehearsal opportunities required.

Lucas and her colleagues (2008) suggest the creation of a language-instruction course for prospective teachers, “namely, one devoted to teaching ELLs and one that all preservice teachers are required to take” (p. 370). The university at which this study was conducted has such a course regarding teaching linguistically diverse students, and it is required for all undergraduates and graduate students in the education school. However, there is no mandatory sequence, making the knowledge base of course participants varied. Additionally, some preservice teachers commented in their interviews that the course pays very little attention to the language demands of mathematics, which has caused them great frustration and often left the participants considering the course to be wasteful.

Gebhard and her peers (2002) report, “teacher education programs have not kept pace with the unprecedented demand for teachers who know how to work effectively with students of linguistically and culturally diverse backgrounds” (p. 221). The secondary methods instructor underwent professional development in language acquisition as background knowledge for altering her course with the content-and-language intervention. In addition, she worked with the director of the content-and-language study, an expert in the field of language-acquisition, in order to hone the intervention. With this model in mind, it seems that converse may be an appropriate approach as well. Perhaps professional development also is needed for the instructors of the teaching ELLs course regarding how to better and more equally incorporate more technical content areas such as mathematics. Following this training, the teacher educator should consult with content-area expert, or perhaps even collaborate with the educator in a co-teaching capacity for the course. Such

alterations to the teaching ELLs course would better serve the students, enrich the knowledge base of the teacher educator, and serve as a realistic option for the university's teacher education program.

Teacher Placement

Connected to the issue of ongoing practice was the issue of the pre-practicum placements of the course enrollees. According to Lucas and her peers (2008), "Teacher education programs can also prepare prospective teachers to teach ELLs by requiring them to spend time in schools and classrooms where they will have contact with ELLs during fieldwork course and fieldwork requirements in regular courses" (p. 370). Such immersion would provide the preservice teachers the linguistic and cultural diversity necessary to challenge their beliefs and instructional practices and, therefore, enrich content (Bartolomé, 2002; Brisk et al., 2002). Only three of the five case-study participants in 2010 were at schools wherein they would be guaranteed this kind of experience. For the 2009 cohort, only two of the five were placed in ELL-rich schools.

As Bryce indicated in his post-semester interview, he did not feel that he needed to focus his energy on language accommodations for the sake of his students, but rather only for the list of requirements for the course and the Practicum Office, failing to recall that strategies for linguistically diverse students can be helpful to all students. Dong (2004) describes a program that requires "field experiences, reflections, reading and discussions" for its secondary preservice teachers, from which the class discussions about the participants field experiences seemed to be crucial to their development as LRP teachers. Maxwell-Jolly and Gándara (2002) add that the programs that offer multicultural infusion in coursework *and* immersive fieldwork "offer a strong possibility of successfully preparing

effective teachers for diverse learners” (p. 61). How much richer the experience may have been for the secondary mathematics preservice teachers in this study if they were all able to “border cross” (Bartolomé, 2002) and have contact with ELLs during their simultaneous pre-practica, returning to class discussions with this new data. These dialogues could prove critical to future teachers development of critical self-awareness of their beliefs and presumptions about teaching in linguistically and culturally diverse settings (Gebhard et al., 2002). Therefore, the school placement of preservice instructors bears consideration for practicum offices not just at this university, but also similar departments across the country.

Limitations of the Study

There were a number of limitations to the design of this study. The course’s number of enrollees was less than 20 people in both 2009 and 2010, which is the typical population of the course at the university but neither adequately represents the current population of preservice teachers nor practicing secondary mathematics teachers. Another limitation was the withdrawal of case-study participation. In the 2010, one preservice teacher decided not to continue with the study and another decided to change majors altogether. In addition, three of the participants did not submit Academic Language Projects, which proved to be plentiful resource for some the coding citations. A third restraint was that only some of the participants were interviewed, but due to the timeframe of a dissertation, as well as the time involved in scheduling meetings with 13-16 people, this was not a doable task. Finally, the researcher was not able to observe pupil impact of the lessons submitted by the case-study participants; a necessary next step in this line of research.

Recommendations for Future Research

A study that measures the impact of an intervention in the field of education seems incomplete without determining the long-term effects of the process – the ultimate goal. Beykont (2002) advocates, “Teacher preparation programs and ongoing professional development can assist teachers who are themselves products of and contributors to the mainstream culture by offering structured and sustained support” (p. xxiv). Thus, one of the recommendations for future research would be to follow the case-study participants into their full practica and their initial years of classroom practice. This would afford the opportunity to not only offer continued coaching regarding the strategies promoted in the content-and-language integrated intervention and grasp more fully how the teachers’ instructional approaches change over time, but also offer an antidote for the lack of pupil impact measure in this study as one would be able to collect student data as well.

Considering another of the study’s limitation – small sample size – another recommendation for future research would be to replicate the study across multiple universities for added statistical power. Potential institutions would need to be identified, and resident faculty and research assistants may require professional development, “as those who teach future teachers... need to develop out knowledge and skills related to the education of ELLs... before we can make the needed changes in the curriculum and in our pedagogy” (p. 370). The faculty, including the methods course instructor, had undergone such training in prior years to this study in addition to her own research interest in literacy in mathematics, and the researcher was apprenticed by the Content Academic Language (CAL) study director, an expert in the realm of language acquisition and multilingual

education. Such development was crucial in the development of the intervention and the study's methodology, but also vital to the interpretation of the data.

The connection of this dissertation to the current discussion systemic functional linguistics (SFL; Halliday, 1985) cannot be ignored, as while strategies were taught for providing access to and teaching the language, identifying what the language of mathematics actually is, as well as its various forms, was not taught. Moschkovich (2004) states, "Although an emphasis on vocabulary may have been sufficient in the past, this perspective does not include current views of what it means to learn mathematics" (p. 192). Lucas and her colleagues (2008) further explicate that this recognition "involves identifying the key vocabulary that students must have in order to have access to curriculum content, understanding the semantic and syntactic complexity of the language use in written instructional materials, and knowing the ways in which students are expected to use language to complete each learning task" (p. 367). Figure 7.1 proposes a way to dissect the mathematics register into elements based on literary constructs, the functions of those constructs, and finally a means through which they may be taught in relation to the conceptual framework of this dissertation.

Table 7.1. *A Possible Academic-Language Lens for Mathematics*

Linguistic constructs	Language functions (Martin & Rose, Zwiers)	Components of Language Acquisition		
		Input	Engagement	Output
Words (symbolic representations)	<ul style="list-style-type: none"> • Description • Comparison 	<ul style="list-style-type: none"> • Symbolic ↔ verbal forms • Provide definitions • Provide connections to real-world, other content areas • Consider word-level structures (adapted from Homza & O'Connor) <ul style="list-style-type: none"> ○ Morphology ○ Cognates ○ Nominalization ○ Multi-semiotic ○ Homonyms/homophones 	<ul style="list-style-type: none"> • Directives: define • Visual models (adapted from Albert): <ul style="list-style-type: none"> ○ Frayer ○ 4-square ○ Venn diagram ○ Semantic map ○ Characteristic & identification tables/diagrams • Other models <ul style="list-style-type: none"> ○ Logs ○ Analogies ○ Songs & poetry 	<ul style="list-style-type: none"> • Definitions • Diagrams • Connections to real-world, other content areas
Phrases (expressions)	<ul style="list-style-type: none"> • Description • Interpretation 	<ul style="list-style-type: none"> • Symbolic ↔ verbal forms • Preposition use • Dense noun phrases (Schleppegrell) • Symbols – operational, grouping 	<ul style="list-style-type: none"> • Directives: translate, simplify, evaluate, compute, calculate, add, subtract, multiply, divide • Other models: <ul style="list-style-type: none"> ○ Logs ○ Songs & poetry ○ Pneumonics 	<ul style="list-style-type: none"> • Translations between symbolic & verbal expressions • Simplified & evaluated expressions
Sentences (relationships)	<ul style="list-style-type: none"> • Description-explanation • Comparison • Interpretation • Procedure 	<ul style="list-style-type: none"> • Symbolic ↔ verbal ↔ graphical forms • Dense noun phrases • Prepositional phrases • Conjunctions (part Schleppegrell) • Symbols – operational, grouping, representational • Algorithms 	<ul style="list-style-type: none"> • Directives: solve, find the solution, describe, determine • Visual models: <ul style="list-style-type: none"> ○ Characteristic & identification tables/diagrams ○ Hierarchical & sequential diagrams • Other models: <ul style="list-style-type: none"> ○ Anticipation guide ○ Logs/journals 	<ul style="list-style-type: none"> • Graphs • Sentence stems • Presentations • Paragraphs

Table 7.1 (*continued*)

Linguistic constructs	Language functions (Martin & Rose, Zwiers)	Components of Language Acquisition		
		Input	Engagement	Output
Paragraphs (text-based & contextual problems)	<ul style="list-style-type: none"> • Interpretation • Persuasion • Cause & effect • Perspective taking • Description-explanation • Comparison • Procedure 	<ul style="list-style-type: none"> • Symbolic ↔ verbal ↔ graphical forms • Prepositional phrases • Conjunctions • Word clusters • Embedded clauses • Multi-semiotic terms/phrases & implied relationships or operations • Idioms • Consider sentence-level/paragraph structure (adapted from Homza & O'Connor) <ul style="list-style-type: none"> ○ Begin with a thesis statement ○ Follow with sentences that include computational evidence 	<ul style="list-style-type: none"> • Directives: explain, justify, determine, how/why/what questions • Visual models: <ul style="list-style-type: none"> ○ Hierarchical or process graphic organizers ○ Flow charts ○ Characteristic & identification tables/diagrams ○ • Other models: <ul style="list-style-type: none"> ○ Anticipation guide ○ Journals 	<ul style="list-style-type: none"> • Symbolic & graphical models • Paragraphs • Essays • Debates • Presentations

As the integration of mathematics pedagogy and language instruction continued to be investigated, the notion of the existence of *genres* of language within mathematics deserves closer examination as well. These categories may offer several layers of language: 1) mathematics as a content-area genre; 2) language uses within the subject, e.g. equations, story/word problem, sentence translations, and coordinate-plane graphs; and 3) subject-specific course as genres, e.g. Geometry, Algebra I, Calculus. Torres-Velasquez and Lobo (2004/5) support the first and third areas, in particular:

“Statistics and data analysis carry their own vocabulary and mathematical language...Teachers of mathematics recognize that the development of number sense involves a lot of experiences working with numbers and numerical concepts, bridging concrete objects to abstract representations. In the same way, students need a lot of experiences working with, thinking about, and talking about real data and their representation in order to develop their *graph sense*” (p. 252).

Dong (2004) adds that secondary teachers need to not only be aware of language acquisition strategies, but also how language functions in their content areas. Expanding or further defining the mathematics register within the SFL paradigm could be a major step in providing mathematics teachers with this necessary knowledge base, especially as “Reading textbooks and solving traditional word problems are... no longer the best examples of how language and learning mathematics intersect” (Moschkovich, 2004, p. 193). Future research should consider such characterization going forward in order to guide those new to the profession into language-instruction proficiency in mathematics.

Closing Comments

One of the questions that the director of the larger content-and-language study and I would periodically ponder is, “Is it too soon for the preservice teachers to consider language instruction or is it enough for them to develop a level of heightened sensitivity to the needs of ELLs early in their careers?” We agree with Richardson (2002) about the amount that preservice teachers are learning in their few short years in higher education; however, we both quickly resolved that the shift has to start from somewhere! In addition, we came to realize that just the intervention is not enough. The prospective teachers need to see language in action in mathematics everywhere they go! Even in attempting to teach this paradigm to these new instructors, we were going against the ways in which they were taught mathematics and we were saying, in essence, those ways were not sufficient to provide access to the subject for all students. However, this conclusion is the most honest statement we could make.

Those of us who teach secondary mathematics are typically the former pupils for whom mathematics simply made sense, and while we may not have been instructed in innovative ways, we possess the innate capacity to make connections and make sense of the content for ourselves. Such students are not those who are failing state exams and being told that they cannot graduate, or dropping out of school because they cannot adjust to how the American school operates nor determine how to navigate the “system.” When we can come to a place of acknowledging this reality, then education becomes more than a job or career, but a mission. This study opened my eyes to another level of social justice that must be acknowledged and dealt with. The mission – if we choose to accept it – is to provide opportunities for every child to access and learn mathematics in a rich and

meaningful way. I have accepted this task and look forward to collaborating with others in the future who have done the same.

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Appendix A:
Fall 2009 Pre-Survey

Content and Language Study for Mathematics
Participant Survey
September 14, 2009

I. Demographic Info

1. Name: _____ Gender: _____
2. Preferred e-mail address: _____
3. Degree Program: (*Circle one.*) Undergraduate Graduate
 - Undergraduate students:
 1. Which pre-practicum field experiences have you completed? (*Circle all that apply.*)
P1 P2 P3
 2. In which practicum experience will you be engaged next semester? (*Circle one.*)
P1 P2 P3 Full Practicum
 3. Please list the name of your practicum site, if known: _____

 - Graduate students:
 1. Which degree are you pursuing? (*Circle one.*) M.A.T. M.Ed.
 2. In which practicum experience will you be engaged next semester? (*Circle one.*)
P1 Full Practicum
 3. Please list the name of your practicum site, if known: _____

4. Are you fluent in any other another languages? (*Circle one.*) Yes No
If so, list the language(s) in which you are proficient: _____
5. Have you lived in a country or region wherein the primary language was not English?
(*Circle one.*) Yes No
If so, please list the country or region and the length of time in which you live there. _

II. Teacher Beliefs Survey

We want to know what you think about mathematics and the place of language and literacy in the mathematics classroom. Please answer all of the following questions as well as you can. There are no “correct” answers, so please be honest about what you think. For each question, circle the number under the answer that best describes your opinion or position. If you have any difficulty understanding any of the questions, ask the survey administrator for assistance.

1. I am familiar and comfortable with planning content and language objectives for my classes.

Strongly disagree	Disagree	Agree	Strongly agree
1	2	3	4

2. When entering mainstream mathematics classes, ELL students may need more time to complete coursework.

Strongly disagree	Disagree	Agree	Strongly agree
1	2	3	4

3. I feel confident in my ability to instruct all students, especially ELL students, in how to interpret the language of mathematics and how to communicate within this language effectively.

Strongly disagree	Disagree	Agree	Strongly agree
1	2	3	4

4. The inclusion of ELL students in mainstream mathematics classes benefits all students.

Strongly disagree	Disagree	Agree	Strongly agree
1	2	3	4

5. Mathematics is not needed in everyday life.

Strongly disagree	Disagree	Agree	Strongly agree
1	2	3	4

6. There is always a particular process or rule to follow in solving a mathematics problem.

Strongly disagree	Disagree	Agree	Strongly agree
1	2	3	4

7. ELL students should avoid using their native languages in mainstream mathematics classes.

Strongly disagree	Disagree	Agree	Strongly agree
1	2	3	4

8. I would welcome the inclusion of ELL students in the mathematics classes that I teach.

Strongly disagree	Disagree	Agree	Strongly agree
1	2	3	4

9. When entering mainstream mathematics classes, ELL students benefit from modified instructional methods to help them understand the content.

Strongly disagree	Disagree	Agree	Strongly agree
1	2	3	4

10. Many mathematics courses could be improved by including objectives and instruction that focus on language development.

Strongly disagree	Disagree	Agree	Strongly agree
1	2	3	4

11. There is only one right method to solve most mathematics problems.

Strongly disagree	Disagree	Agree	Strongly agree
1	2	3	4

12. I have adequate training to work with ELL students in a secondary mathematics classroom.

Strongly disagree	Disagree	Agree	Strongly agree
1	2	3	4

13. Using language-learning and literacy strategies in mathematics is a bad idea.

Strongly disagree	Disagree	Agree	Strongly agree
1	2	3	4

14. Mathematics is a fluid subject wherein you can be creative and discover things for yourself.

Strongly disagree	Disagree	Agree	Strongly agree
1	2	3	4

15. ELL students should be included in mainstream mathematics classes only when they have attained at least moderate fluency in English.

Strongly disagree	Disagree	Agree	Strongly agree
1	2	3	4

16. ELL students should be able to use their native languages in any class.

Strongly disagree	Disagree	Agree	Strongly agree
1	2	3	4

17. Teachers should not modify their instruction for the ELL students enrolled in mainstream mathematics classes.

Strongly disagree	Disagree	Agree	Strongly agree
1	2	3	4

18. ELL students should not require more time than other students to complete their mathematics assignments.

Strongly disagree	Disagree	Agree	Strongly agree
1	2	3	4

19. I am interested in receiving more training in working with ELL students in mathematics classes.

Strongly disagree	Disagree	Agree	Strongly agree
1	2	3	4

20. ELL students should have a separate mathematics class.

Strongly disagree	Disagree	Agree	Strongly agree
1	2	3	4

21. I am not sure how to teach any student about the linguistic and communicative aspects of mathematics.

Strongly disagree	Disagree	Agree	Strongly agree
1	2	3	4

22. Language development should only be taught within English courses.

Strongly disagree	Disagree	Agree	Strongly agree
1	2	3	4

23. As a mathematics teacher, I intend to concentrate exclusively on mathematics-content objectives for my classes.

Strongly disagree	Disagree	Agree	Strongly agree
1	2	3	4

24. Language-learning strategies and literacy strategies are useful in all kinds of classes.

Strongly disagree	Disagree	Agree	Strongly agree
1	2	3	4

25. There are many different ways to solve most mathematics problems.

Strongly disagree	Disagree	Agree	Strongly agree
1	2	3	4

26. Learning mathematics involves mostly memorizing.

Strongly disagree	Disagree	Agree	Strongly agree
1	2	3	4

27. It's important to learn mathematics because mathematics knowledge is useful in everyday life.

Strongly disagree	Disagree	Agree	Strongly agree
1	2	3	4

28. In mathematics, problem can be solved without using any particular algorithms or rules.

Strongly disagree	Disagree	Agree	Strongly agree
1	2	3	4

III. Mathematics Experience Demonstration

Quietly and on your own, think about your experience as a mathematics student in a secondary classroom. Visualize yourself walking into a school, down the hallway, and into your mathematics class. Consider what students are doing in the classroom, where the teacher is located, what kinds of materials or tools the teacher and/or students are using, and what concepts are being discussed in the class.

- After thinking about these things for a few minutes, draw a picture of this classroom in action. Use colored pencils, crayons, or markers to create your drawing, but do not be concerned about your artistic abilities. ☺
- After you have drawn your picture, write 1-2 paragraphs describing what the teacher and students are doing in the classroom.

Appendix B:
Fall 2009 Post-Survey

Content and Language Study for Mathematics
Participant Survey
December 7, 2009

I. Demographic Info

1. Name: _____
2. Preferred e-mail address: _____
3. Please list the name of your Spring 2010 practicum site, if known:

II. Teacher Beliefs Survey

We want to know what you think about mathematics and the place of language and literacy in the mathematics classroom. Please answer all of the following questions as well as you can. There are no "correct" answers, so please be honest about what you think. For each question, circle the number under the answer that best describes your opinion or position. If you have any difficulty understanding any of the questions, ask the survey administrator for assistance.

1. I am familiar and comfortable with planning content and language objectives for my classes.

Strongly disagree	Disagree	Agree	Strongly agree
1	2	3	4

2. When entering mainstream mathematics classes, ELL students may need more time to complete coursework.

Strongly disagree	Disagree	Agree	Strongly agree
1	2	3	4

3. I feel confident in my ability to instruct all students, especially ELL students, in how to interpret the language of mathematics and how to communicate within this language effectively.

Strongly disagree	Disagree	Agree	Strongly agree
1	2	3	4

4. The inclusion of ELL students in mainstream mathematics classes benefits all students.

Strongly disagree	Disagree	Agree	Strongly agree
1	2	3	4

5. Mathematics is not needed in everyday life.

Strongly disagree	Disagree	Agree	Strongly agree
1	2	3	4

6. There is always a particular process or rule to follow in solving a mathematics problem.

Strongly disagree	Disagree	Agree	Strongly agree
1	2	3	4

7. ELL students should avoid using their native languages in mainstream mathematics classes.

Strongly disagree	Disagree	Agree	Strongly agree
1	2	3	4

8. I would welcome the inclusion of ELL students in the mathematics classes that I teach.

Strongly disagree	Disagree	Agree	Strongly agree
1	2	3	4

9. When entering mainstream mathematics classes, ELL students benefit from modified instructional methods to help them understand the content.

Strongly disagree	Disagree	Agree	Strongly agree
1	2	3	4

10. Many mathematics courses could be improved by including objectives and instruction that focus on language development.

Strongly disagree	Disagree	Agree	Strongly agree
1	2	3	4

11. There is only one right method to solve most mathematics problems.

Strongly disagree	Disagree	Agree	Strongly agree
1	2	3	4

12. I have adequate training to work with ELL students in a secondary mathematics classroom.

Strongly disagree	Disagree	Agree	Strongly agree
1	2	3	4

13. Using language-learning and literacy strategies in mathematics is a bad idea.

Strongly disagree	Disagree	Agree	Strongly agree
1	2	3	4

14. Mathematics is a fluid subject wherein you can be creative and discover things for yourself.

Strongly disagree	Disagree	Agree	Strongly agree
1	2	3	4

15. ELL students should be included in mainstream mathematics classes only when they have attained at least moderate fluency in English.

Strongly disagree	Disagree	Agree	Strongly agree
1	2	3	4

16. ELL students should be able to use their native languages in any class.

Strongly disagree	Disagree	Agree	Strongly agree
1	2	3	4

17. Teachers should not modify their instruction for the ELL students enrolled in mainstream mathematics classes.

Strongly disagree	Disagree	Agree	Strongly agree
1	2	3	4

18. ELL students should not require more time than other students to complete their mathematics assignments.

Strongly disagree	Disagree	Agree	Strongly agree
1	2	3	4

19. I am interested in receiving more training in working with ELL students in mathematics classes.

Strongly disagree	Disagree	Agree	Strongly agree
1	2	3	4

20. ELL students should have a separate mathematics class.

Strongly disagree	Disagree	Agree	Strongly agree
1	2	3	4

21. I am not sure how to teach any student about the linguistic and communicative aspects of mathematics.

Strongly disagree	Disagree	Agree	Strongly agree
1	2	3	4

22. Language development should only be taught within English courses.

Strongly disagree	Disagree	Agree	Strongly agree
1	2	3	4

23. As a mathematics teacher, I intend to concentrate exclusively on mathematics-content objectives for my classes.

Strongly disagree	Disagree	Agree	Strongly agree
1	2	3	4

24. Language-learning strategies and literacy strategies are useful in all kinds of classes.

Strongly disagree	Disagree	Agree	Strongly agree
1	2	3	4

25. There are many different ways to solve most mathematics problems.

Strongly disagree	Disagree	Agree	Strongly agree
1	2	3	4

26. Learning mathematics involves mostly memorizing.

Strongly disagree	Disagree	Agree	Strongly agree
1	2	3	4

27. It's important to learn mathematics because mathematics knowledge is useful in everyday life.

Strongly disagree	Disagree	Agree	Strongly agree
1	2	3	4

28. In mathematics, problems can be solved without using any particular algorithms or rules.

Strongly disagree	Disagree	Agree	Strongly agree
1	2	3	4

III. Ideal Classroom Demonstration & Reflection

- A. Quietly and on your own, think about the knowledge and experiences you have acquired in this secondary mathematics methods course. Visualize yourself as an instructor, and consider what your ideal mathematics classroom would look like to an observer in your class. What would students be doing in the classroom? Where would you be located? What kinds of materials or tools would you and/or students use? What concepts would be discussed in the class? Also, consider the strategies have you learned from this class to assist you in the teaching of mathematics and in the addressing of academic language in your practice.
- After thinking about these things for a few minutes, draw a picture of your ideal classroom in action. Use colored pencils, crayons, or markers to create your drawing, but again, do not be concerned about your artistic abilities.
 - After you have drawn your picture, write 1-2 paragraphs describing your ideal classroom.

- B. Is there more you would like to learn about working with ELLs in secondary mathematics? Please describe what more you would like to know.

- C. Additional comments:

Appendix C:
Fall 2010 Pre-Survey

Content and Language Study for Mathematics
Participant Survey
September 13, 2010

I. Demographic Info

1. Name: _____ Gender: _____

2. Preferred e-mail address: _____

3. Degree Program: (*Circle one.*) Undergraduate Graduate

- Undergraduate students:

1. In which pre-practicum field experiences are you currently participating?
(*Circle all that apply.*)

P1 P2 P3

2. In which practicum experience will you be engaged next semester? (*Circle one.*)

P1 P2 P3 Full Practicum

3. Please list the name of your practicum site, if known: _____

- Graduate students:

1. Which degree are you pursuing? (*Circle one.*) M.A.T. M.Ed.

2. In which practicum experience will you be engaged next semester? (*Circle one.*)

P1 Full Practicum

3. Please list the name of your practicum site, if known: _____

4. For the following courses, which are part of the Lynch School's Teaching English-language Learners Certificate program, please indicate your experiences or intentions:

a. ED 346 – Teaching Bilingual Students

☐ Not planning to take

☐ Taking it this semester

☐ Taking it another semester. *Please indicate the semester:* _____

☐ Already completed. *Please indicate the semester:* _____

b. ED 621 – Literacy and Bilingualism

☐ Not planning to take

☐ Taking it this semester

☐ Taking it another semester. *Please indicate the semester:* _____

☐ Already completed. *Please indicate the semester:* _____

5. Are you fluent in any other another languages? (*Circle one.*) Yes No

If so, list the language(s) in which you are proficient: _____

6. Have you lived in a country or region wherein the primary language was not English?

(*Circle one.*) Yes No

If so, please list the country or region and the length of time in which you live there. _

II. Teacher Beliefs Survey

We want to know what you think about mathematics and the place of language and literacy in the mathematics classroom. Please answer all of the following questions as well as you can. There are no “correct” answers, so please be honest about what you think. For each question, circle the number under the answer that best describes your opinion or position. If you have any difficulty understanding any of the questions, ask the survey administrator for assistance.

1. I am comfortable with planning mathematics-content objectives for my classes.

Strongly disagree	Disagree	Agree	Strongly agree
1	2	3	4

2. When entering mainstream mathematics classes, ELL students should be given more time to complete coursework.

Strongly disagree	Disagree	Agree	Strongly agree
1	2	3	4

3. I feel confident in my ability to instruct all students, especially ELL students, in how to interpret the language of mathematics and how to communicate within this language effectively.

Strongly disagree	Disagree	Agree	Strongly agree
1	2	3	4

4. The inclusion of ELL students in mainstream mathematics classes benefits all students.

Strongly disagree	Disagree	Agree	Strongly agree
1	2	3	4

5. Mathematics is not needed in everyday life.

Strongly disagree	Disagree	Agree	Strongly agree
1	2	3	4

6. There is always a particular process or rule to follow in solving a mathematics problem.

Strongly	Disagree	Agree	Strongly
----------	----------	-------	----------

disagree			agree
1	2	3	4

7. ELL students should avoid using their native languages in mainstream mathematics classes.

Strongly disagree	Disagree	Agree	Strongly agree
1	2	3	4

8. I would welcome the inclusion of ELL students in the mathematics classes that I teach.

Strongly disagree	Disagree	Agree	Strongly agree
1	2	3	4

9. When entering mainstream mathematics classes, ELL students benefit from modified instructional methods to help them understand the content.

Strongly disagree	Disagree	Agree	Strongly agree
1	2	3	4

10. Many mathematics courses could be improved by including objectives and instruction that focus on language development.

Strongly disagree	Disagree	Agree	Strongly agree
1	2	3	4

11. There is only one right method to solve most mathematics problems.

Strongly disagree	Disagree	Agree	Strongly agree
1	2	3	4

12. I have adequate training to work with ELL students in a secondary mathematics classroom.

Strongly disagree	Disagree	Agree	Strongly agree
1	2	3	4

13. Using language-learning and literacy strategies in mathematics is a bad idea.

Strongly disagree	Disagree	Agree	Strongly agree
1	2	3	4

14. Mathematics is a fluid subject wherein you can be creative and discover things for yourself.

Strongly disagree	Disagree	Agree	Strongly agree
1	2	3	4

15. I believe that ELL students should be included in mainstream mathematics classes only when they have attained native-like fluency in English.

Strongly disagree	Disagree	Agree	Strongly agree
-------------------	----------	-------	----------------

- | | 1 | 2 | 3 | 4 |
|---|------------------------|---------------|------------|---------------------|
| 16. It is my belief that ELL students should be able to use their native languages in any class. | Strongly disagree
1 | Disagree
2 | Agree
3 | Strongly agree
4 |
| 17. Teachers should not modify their instruction for the ELL students enrolled in mainstream mathematics classes. | Strongly disagree
1 | Disagree
2 | Agree
3 | Strongly agree
4 |
| 18. ELL students should not require more time than other students to complete their mathematics assignments. | Strongly disagree
1 | Disagree
2 | Agree
3 | Strongly agree
4 |
| 19. I am interested in receiving more professional development in the future regarding addressing the needs of ELLs in mathematics classes, although I have enough training to begin my practice. | Strongly disagree
1 | Disagree
2 | Agree
3 | Strongly agree
4 |
| 20. ELL students should have a separate mathematics class. | Strongly disagree
1 | Disagree
2 | Agree
3 | Strongly agree
4 |
| 21. I am not sure how to teach any student about the linguistic and communicative aspects of mathematics. | Strongly disagree
1 | Disagree
2 | Agree
3 | Strongly agree
4 |
| 22. Language development should only be taught within English courses. | Strongly disagree
1 | Disagree
2 | Agree
3 | Strongly agree
4 |
| 23. As a mathematics teacher, I intend to concentrate exclusively on mathematics-content objectives for my classes. | Strongly disagree
1 | Disagree
2 | Agree
3 | Strongly agree
4 |
| 24. Language-learning strategies and literacy strategies are useful in all kinds of classes. | | | | |

- | | | | | |
|--|---------------------------|---------------|------------|------------------------|
| | Strongly
disagree
1 | Disagree
2 | Agree
3 | Strongly
agree
4 |
|--|---------------------------|---------------|------------|------------------------|
25. There are many different ways to solve most mathematics problems.
- | | | | | |
|--|---------------------------|---------------|------------|------------------------|
| | Strongly
disagree
1 | Disagree
2 | Agree
3 | Strongly
agree
4 |
|--|---------------------------|---------------|------------|------------------------|
26. Learning mathematics involves mostly memorizing.
- | | | | | |
|--|---------------------------|---------------|------------|------------------------|
| | Strongly
disagree
1 | Disagree
2 | Agree
3 | Strongly
agree
4 |
|--|---------------------------|---------------|------------|------------------------|
27. It's important to learn mathematics because mathematics knowledge is useful in everyday life.
- | | | | | |
|--|---------------------------|---------------|------------|------------------------|
| | Strongly
disagree
1 | Disagree
2 | Agree
3 | Strongly
agree
4 |
|--|---------------------------|---------------|------------|------------------------|
28. In mathematics, problems can be solved without using a particular algorithm or rule.
- | | | | | |
|--|---------------------------|---------------|------------|------------------------|
| | Strongly
disagree
1 | Disagree
2 | Agree
3 | Strongly
agree
4 |
|--|---------------------------|---------------|------------|------------------------|
29. I am comfortable with planning language objectives for my classes.
- | | | | | |
|--|---------------------------|---------------|------------|------------------------|
| | Strongly
disagree
1 | Disagree
2 | Agree
3 | Strongly
agree
4 |
|--|---------------------------|---------------|------------|------------------------|
30. I do not have enough education regarding working with ELLs to begin working with them in the mathematics classes I will teach and need more training.
- | | | | | |
|--|---------------------------|---------------|------------|------------------------|
| | Strongly
disagree
1 | Disagree
2 | Agree
3 | Strongly
agree
4 |
|--|---------------------------|---------------|------------|------------------------|
31. I am still struggling with determining content objectives for the mathematics classes that I teach.
- | | | | | |
|--|---------------------------|---------------|------------|------------------------|
| | Strongly
disagree
1 | Disagree
2 | Agree
3 | Strongly
agree
4 |
|--|---------------------------|---------------|------------|------------------------|

Name: _____

III. Mathematics Experience Demonstration

Quietly and on your own, think about your experience as a mathematics student in a secondary classroom. Visualize yourself walking into a school, down the hallway, and into your mathematics class. Consider what students are doing in the classroom, where the teacher is located, what kinds of materials or tools the teacher and/or students are using, and what concepts are being discussed in the class.

1. After thinking about these things for a few minutes, choose a mathematics topic and draw a picture of this classroom during instruction. You may use colored pencils, crayons, or markers to create your drawing, but do not be concerned about your artistic abilities. ☺
2. After you have drawn your picture, write 1-2 paragraphs describing what the teacher and students are doing in the classroom, how the mathematics is being taught, and how an ELL would function in this kind of classroom.

Appendix D:
Fall 2010 Post-Survey

Content and Language Study for Mathematics
Participant Survey
December 6, 2010

I. Demographic Info

1. Name: _____
2. Preferred e-mail address: _____
3. Please list the name of your Spring 2010 practicum site, if known:

II. Ideal Classroom Demonstration & Reflection

Quietly and on your own, think about the knowledge and experiences you have acquired in this secondary mathematics methods course. Visualize yourself as an instructor, and consider what your ideal mathematics classroom would look like to an observer in your class. What would students be doing in the classroom? Where would you be located? What kinds of materials or tools would you and/or students use? What concepts would be discussed in the class? Also, consider the strategies have you learned from this class to assist you in the teaching of mathematics and in the addressing of academic language in your practice.

- After thinking about these things for a few minutes, draw a picture of your ideal classroom in action. Use colored pencils, crayons, or markers to create your drawing, but again, do not be concerned about your artistic abilities.
- After you have drawn your picture, write 1-2 paragraphs describing your ideal classroom.
(Feel free to use the back of this page, if needed.)

Name: _____

III. Teacher Beliefs Survey

We want to know what you think about mathematics and the place of language and literacy in the mathematics classroom. Please answer all of the following questions as well as you can. There are no “correct” answers, so please be honest about what you think. For each question, circle the number under the answer that best describes your opinion or position. If you have any difficulty understanding any of the questions, ask the survey administrator for assistance.

1. I am comfortable with planning mathematics-content objectives for my classes.

Strongly disagree	Disagree	Agree	Strongly agree
1	2	3	4

2. When entering mainstream mathematics classes, ELL students should be given more time to complete coursework.

Strongly disagree	Disagree	Agree	Strongly agree
1	2	3	4

3. I feel confident in my ability to instruct all students, especially ELL students, in how to interpret the language of mathematics and how to communicate within this language effectively.

Strongly disagree	Disagree	Agree	Strongly agree
1	2	3	4

4. The inclusion of ELL students in mainstream mathematics classes benefits all students.

Strongly disagree	Disagree	Agree	Strongly agree
1	2	3	4

5. Mathematics is not needed in everyday life.

Strongly disagree	Disagree	Agree	Strongly agree
1	2	3	4

6. There is always a particular process or rule to follow in solving a mathematics problem.

Strongly disagree	Disagree	Agree	Strongly agree
1	2	3	4

7. ELL students should avoid using their native languages in mainstream mathematics classes.

Strongly disagree	Disagree	Agree	Strongly agree
1	2	3	4

8. I would welcome the inclusion of ELL students in the mathematics classes that I teach.

Strongly disagree	Disagree	Agree	Strongly agree
1	2	3	4

9. When entering mainstream mathematics classes, ELL students benefit from modified instructional methods to help them understand the content.

Strongly disagree	Disagree	Agree	Strongly agree
1	2	3	4

10. Many mathematics courses could be improved by including objectives and instruction that focus on language development.

Strongly disagree	Disagree	Agree	Strongly agree
1	2	3	4

11. There is only one right method to solve most mathematics problems.

Strongly disagree	Disagree	Agree	Strongly agree
1	2	3	4

12. I have adequate training to work with ELL students in a secondary mathematics classroom.

Strongly disagree	Disagree	Agree	Strongly agree
1	2	3	4

13. Using language-learning and literacy strategies in mathematics is a bad idea.

Strongly disagree	Disagree	Agree	Strongly agree
1	2	3	4

14. Mathematics is a fluid subject wherein you can be creative and discover things for yourself.

Strongly disagree	Disagree	Agree	Strongly agree
1	2	3	4

15. I believe that ELL students should be included in mainstream mathematics classes only when they have attained native-like fluency in English.

Strongly disagree	Disagree	Agree	Strongly agree
1	2	3	4

16. It is my belief that ELL students should be able to use their native languages in any class.

Strongly disagree	Disagree	Agree	Strongly agree
1	2	3	4

17. Teachers should not modify their instruction for the ELL students enrolled in mainstream mathematics classes.

Strongly disagree	Disagree	Agree	Strongly agree
1	2	3	4

18. ELL students should not require more time than other students to complete their mathematics assignments.

Strongly disagree	Disagree	Agree	Strongly agree
1	2	3	4

19. I am interested in receiving more professional development in the future regarding addressing the needs of ELLs in mathematics classes, although I have enough training to begin my practice.

Strongly disagree	Disagree	Agree	Strongly agree
1	2	3	4

20. ELL students should have a separate mathematics class.

Strongly disagree	Disagree	Agree	Strongly agree
1	2	3	4

21. I am not sure how to teach any student about the linguistic and communicative aspects of mathematics.

Strongly disagree	Disagree	Agree	Strongly agree
1	2	3	4

22. Language development should only be taught within English courses.

Strongly disagree	Disagree	Agree	Strongly agree
1	2	3	4

23. As a mathematics teacher, I intend to concentrate exclusively on mathematics-content objectives for my classes.

Strongly disagree	Disagree	Agree	Strongly agree
1	2	3	4

24. Language-learning strategies and literacy strategies are useful in all kinds of classes.

Strongly disagree	Disagree	Agree	Strongly agree
1	2	3	4

25. There are many different ways to solve most mathematics problems.

Strongly disagree	Disagree	Agree	Strongly agree
1	2	3	4

26. Learning mathematics involves mostly memorizing.

Strongly disagree	Disagree	Agree	Strongly agree
1	2	3	4

27. It's important to learn mathematics because mathematics knowledge is useful in everyday life.

Strongly disagree	Disagree	Agree	Strongly agree
1	2	3	4

28. In mathematics, problems can be solved without using a particular algorithm or rule.

Strongly disagree	Disagree	Agree	Strongly agree
1	2	3	4

29. I am comfortable with planning language objectives for my classes.

Strongly disagree	Disagree	Agree	Strongly agree
1	2	3	4

30. I do not have enough education regarding working with ELLs to begin working with them in the mathematics classes I will teach and need more training.

Strongly disagree	Disagree	Agree	Strongly agree
1	2	3	4

31. I am still struggling with determining content objectives for the mathematics classes that I teach.

Strongly disagree	Disagree	Agree	Strongly agree
1	2	3	4

IV. Additional Questions *(Feel free to use the back for more space if needed.)*

A. The following is a list of some of the concepts and strategies taught during this course regarding working with ELLs in secondary mathematics.

- Writing language objectives
- Comprehensible input/output
- Collaborative learning
- Linking language to student background (culture, language, etc.)
- Use of manipulatives, hands-on materials
- Linking language to prior mathematics
- Assessment options
- Teaching vocabulary (“bricks” and “mortar”)

Choose a strategy or concept that you plan to implement or consider in your instruction of ELLs in your next teaching setting. Why does this strategy or concept stand out for you?

B. Is there more you would like to learn about working with ELLs in secondary mathematics? Please describe what more you would like to know.

C. Please comment on anything you deem important for the researchers to know going forward (i.e. other suggestions or any criticisms regarding the “infusion” of academic-language pedagogy within this course as well as support you believe you would need beyond this term).

Appendix E:
Fall 2009 & 2010 Pre-Interview Protocol

Pre-Interview Questions

I. Language/Cross-Cultural experience

- 1) What is/are your native language(s)?
- 2) Are you fluent in any other languages?
If yes, ask the following:
 - a. How would you rate your proficiency level in each of your additional language(s)?
 - b. How would you categorize your level of fluency in each language? (i.e. Beginner, early intermediate, intermediate, advanced/proficient, native fluency)
 - c. What are your capabilities in each language? (e.g. order a meal, read a book, engage in social interactions, write an essay, etc.)
- 3) Have you visited a place wherein the primary language was not English?
 If so, what was the nature of that experience? What was the length of time for the experience?
- 4) Have you studied in a place wherein the primary language was not English?
 If so, what was the nature of that experience (i.e. undergraduate semester abroad, high school exchange program)? What was the length of time for the experience?
- 5) Have you worked in a place wherein the primary language was not English?
 If so, what was the nature of that experience? What was the length of time for the experience?

II. Teaching experience

- 1) Have you had previous teaching experience in a school setting?
 If so, what was the nature of that experience? (e.g. student teaching, teaching assistant, after school program, classroom teacher, tutoring, etc.)
- 2) If you have taught before, at what instructional level? (e.g. elementary, middle, secondary, higher education, adult education)
- 3) Have you received training in teaching language-minority/ELL students?
 If so, what the type of training you have received? (e.g. in-service workshop, college coursework.)
- 4) Have you previously worked with English language learners?
 If so, in what capacity? (e.g. as a teacher, teaching assistant, volunteer, camp counselor)

III. Additional questions

- 1) What do you think could be the benefits of including ELL students in a mathematics class?
- 2) What challenges do you think could arise when including ELL students in a mathematics class?
- 3) What would like to know about working with ELLs in mathematics classroom?

Appendix F:
Fall 2009 & 2010 Post-Interview Protocol

Post-Interview Questions

Overall impressions of pre-practicum experience:

1. How are you feeling about teaching mathematics, now that you've been through your first (P1/grads) or second (P3s) semester in the classroom? Was there anything unexpected about the experience for you?
2. Can you describe what your experience was like working with ELL/diverse students in your classes?
3. What strategies/interactions did you notice your cooperating teacher using for ELL students to further instruction? Did you try to employ/duplicate these as well?
4. How many lessons did you teach? Did you feel they were successful mathematically? Language-wise?
5. What concepts did you teach this term? Can you describe the kinds of changes you made to your instruction in order to consider ELL learners? Why?

Overall secondary mathematics methods course impressions:

1. Thinking of the content of the secondary mathematics methods course, describe the themes that stuck out most to you. Why?
2. Did you feel that the mathematics methods/strategies discussed enhanced your instruction? What about the language-based/ELL strategies?
3. Regarding the systems of equations "What If...":
 - a. How did you determine which vocabulary words to discuss (Edith included "mortars" – *system, row, column*)?
 - b. Why do you feel that having ELL students restate the problem is important?
 - c. You mentioned "simplifying" the language. How exactly would you do this? (Edith & Grace so far)
 - d. Did you employ these strategies in your pre-prac lessons? How successful did they prove to be?
 - e. Would there be room for multiple methods of solving the system? How might this benefit ELLs?
4. What do you think are components good language objectives?
5. Is there anything regarding the instruction of ELLs that you feel the class could have better prepared you for?
6. Can you expound upon the kinds of strategies/activities you're looking for in any future ELL training? (per surveys – Edith/Grace)
7. What kind of experiences are you looking for in the spring with ELL students?

From personal artifacts (Seth):

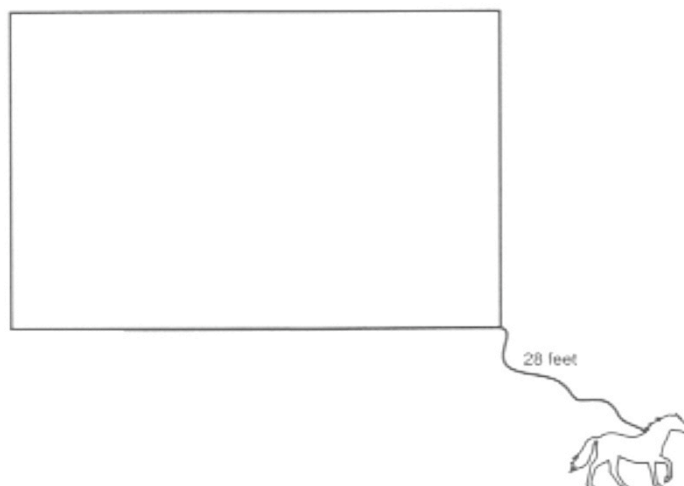
- a. In the case study assignment, you stated that Ms. Elmore used strategies and classroom setup to aid her ELL/special needs students. For strategies, you mentioned 1) "embedding language within a meaningful context," 2) "modifying language," 3) paraphrasing and repetition ("judiciously"), and 4) negotiating meaning. Can you expound upon what these mean to you? Did you incorporate these into your own lessons? What about Ms. E's classroom setup supported ELL students?

- b. You also stated, “Language provides a realm of analytic learning that students must access at every level and in every subject matter.” Can you expound upon what you meant by this? How would this help ELL students?
- c. In your newsletter, you highlighted Destination Math (computer program) as a helpful too for ELLs with its “mix of narratives and word problems that work to help students with the language.” Can you expound upon what you meant by this and give specific examples that would help ELL students?
- d. Regarding the ELL student that you mentioned, do you recall what she needed help with in your lesson? You said that a visual representation could have helped her. Is that how you assisted her with the task? If so, how did this also “foster her language development?” At what point was she able to grasp the academic language that you were using?
- e. Did you employ group work in all of your lessons? Did the CT do so as well? (Mentions that school embraces Vygotsky ~ social learning...)
- f. What kind of feedback did you received on the exit slips, specifically from the ELL students?
- g. What were your language objectives for this lesson? How were they accomplished?
- h. You mentioned a need to address “seemingly subtle needs of the [ELL] students.” Can you expound upon what you meant by this? Offer an example?
- i. In the systems of equations “What If,” you mentioned providing an example problem with the “actual” and “simplified” language noted. Can you give me an example of this? How would this be helpful to an ELL student?
- j. In the MTQ (layers) “What if,” how did you determine the vocabulary you wanted to highlight (folding, layers, functions) for ELLs?
- k. You did not mention any ways that graphing calculators can assist ELL students on that “What If.” Do you have any thoughts now?

Assessment/Discussion:

What do you think would be the challenges for ELL/diverse students in solving this problem? How would you assist diverse learners in addressing this problem?

A rectangular barn measures 50 feet by 100 feet. At one corner, a horse is tethered with a 28-foot rope, as shown in the figure. On how many square feet of grass is the horse able to graze? What if the dimensions of the barn were 60 feet by 150 feet?



Appendix G:
Initial Coding Frequency Tables

Table 1. *Analysis of Comprehensible Input Coding*

Code	Artifacts				
	Critical Analysis	Case Study Analysis	Imagine Ifs	Lesson Organizer & Reflection	Academic Language Project
Everyday language to define math terms		Natalie (1)		Scott (1)	Bryce (1) Mabel (7)
Everyday words with different meanings in math				Bryce (2) Mabel (1)	
Language instruction in math		Mabel (1)		Bryce (2) Mabel (1)	Bryce (9) Mabel (7)
Language objectives					Bryce (5) Mabel (6)
Multiple intelligences		Bryce (1) Mabel (1)	Natalie (2)	Scott (1)	
Multiple representations	Mabel (2)	Mabel (2) Natalie (1)	Bryce (3) Hank (1) Mabel (1) Scott (1)		
Placement of activity within unit		Bryce (1) Hank (1) Mabel (2) Natalie (1) Scott (1)			
Possible language accommodation	Mabel (2)	Bryce (7) Hank (1) Mabel (1)	Bryce (1) Hank (3) Scott (1)	Bryce (4) Mabel (6) Natalie (4)	Bryce (12) Mabel (17)
Prior knowledge of students		Natalie (1)	Mabel (2)	Bryce (1) Hank (2) Mabel (3) Scott (2)	Bryce (3) Mabel (5)
Real-world connections	Hank (1) Mabel (1) Scott (3)	Natalie (2) Scott (4)	Hank (1) Mabel (1) Scott (1)	Mabel (2) Scott (1)	Bryce (3) Mabel (8)
Scaffolding content	Mabel (3) Natalie (1)	Bryce (1) Natalie (3) Scott (1)	Bryce (2) Hank (2) Natalie (1) Scott (2)	Bryce (3) Hank (1) Mabel (6) Natalie (1) Scott (3)	Bryce (1) Mabel (3)

Table 1. (*continued*)

Code	Artifacts				
	Critical Analysis	Case Study Analysis	Imagine Ifs	Lesson Organizer & Reflection	Academic Language Project
Scaffolding language	Mabel (2) Natalie (1)	Natalie (1) Scott (1)	Bryce (1) Hank (1) Mabel (1) Natalie (1) Scott (2)	Hank (1) Mabel (3) Natalie (1) Scott (1)	Mabel (4)
Specific tools for language instruction	Mabel (1)		Bryce (1)	Bryce (2) Mabel (1)	Bryce (3) Mabel (4)
Symbolic language focus					Mabel (4)
Variety in instructional methods	Bryce (2) Mabel (1) Scott (1)	Bryce (3) Mabel (1) Natalie (1) Scott (1)	Bryce (3) Natalie (1) Scott (1)	Bryce (1) Scott (3)	Bryce (4)
Variety in pacing			Bryce (2)		
Vocabulary focus		Mabel (1)	Bryce (2) Hank (1) Mabel (2) Scott (1)	Mabel (3)	Bryce (29) Mabel (24)

Table 2. *Analysis of Student Engagement Coding*

Code	Artifacts				
	Critical Analysis	Case Study Analysis	Imagine Ifs	Lesson Organizer & Reflection	Academic Language Project
Communication emphasis	Bryce (1) Mabel (4)	Hank (1) Natalie (1)	Bryce (1) Mabel (2)	Bryce (2) Hank (4) Mabel (1) Natalie (1)	
Cooperative learning	Bryce (2) Mabel (2) Natalie (1)	Bryce (1) Hank (3) Mabel (1) Natalie (1)	Bryce (3) Mabel (3) Natalie (1) Scott (1)	Bryce (4) Hank (2) Mabel (2) Natalie (4) Scott (1)	Bryce (4) Mabel (4)
Evidence of student engagement		Hank (1)		Hank (1) Natalie (1) Scott (1)	
Hands-on investigation or manipulatives	Mabel (1)	Bryce (2) Hank (2) Mabel (3) Natalie (1) Scott (5)	Hank (1) Scott (2)	Natalie (2) Scott (5)	
Heterogeneous grouping		Hank (1) Natalie (3)		Scott (1)	
Pitfalls of cooperative learning	Mabel (1)		Natalie (1) Scott (1)	Bryce (1) Natalie (1)	
Questioning technique		Bryce (4) Hank (1) Mabel (4) Natalie (1) Scott (3)	Scott (1)	Bryce (3) Mabel (2) Scott (4)	Bryce (11) Mabel (21)
Role of student effort		Scott (2)			
Role of student ownership			Hank (1) Natalie (2)		
Talk moves			Scott (1)		
Teacher as facilitator		Bryce (1)		Mabel (1) Natalie (1) Scott (1)	

Table 3. *Analysis of Comprehensible Output Coding*

Code	Artifacts				
	Critical Analysis	Case Study Analysis	Imagine Ifs	Lesson Organizer & Reflection	Academic Language Project
Hypothesis testing function	Mabel (1)	Bryce (1) Hank (1) Mabel (1) Natalie (2) Scott (5)		Bryce (1) Mabel (1) Natalie (2)	Bryce (4) Mabel (23)
Metalinguistic function		Hank (1) Natalie (2) Scott (2)		Bryce (2) Natalie (4) Scott (1)	Bryce (1) Mabel (3)
Noticing-triggering function		Scott (1)		Natalie (2) Scott (3)	Mabel (11)

Appendix H:
Bryce's Facebook-Styled Graphic Organizer

facebook


[View Photos of me \(1,245\)](#)
[View Videos of me \(23\)](#)
[Poke me](#)
Information

Networks:

Functions

Relationship Status:

It's Complicated with
[Polynomial Functions](#)

Birthday:

January 1, 1398

Current City:

Math is everywhere

Rational Functions $f(x) = \frac{p(x)}{q(x)}$

Nothing just makes sense anymore.

Wall

Info

Photos

Questions

Domain Are you still up for hanging out tomorrow night? I can meet anywhere where the polynomial in the denominator is not equal to _____.

[Comment](#)
[Like](#)
[See Friendship](#)

Asymptote Let's not get too touchy-feely. You know that I'm a _____ that you can approach, but never touch.

[Comment](#)
[Like](#)
[See Friendship](#)
Rational Functions is now friends with [Polynomials](#).Rational Functions likes [Math is FUN!!](#)

Vertical Asymptote I guess we could meet up to talk. You can find me at wherever the x -values of the polynomial in the _____ are equal to _____. Just to let you know, me and Domain are getting pretty close, we have so much in common!

[Comment](#)
[Like](#)
[See Friendship](#)

Horizontal Asymptote I can give you my phone number, but that depends on you, Rational Functions. Let n be the degree of the top polynomial, and m be the degree of the bottom.

Here's what happens:

If n _____ m , then I will be _____.If n _____ m , then I will be _____.If n _____ m , then I will be _____.
[Comment](#)
[Like](#)
[See Friendship](#)

Arrow Notation Point me in the right direction! I'm so lost. Normally, I write about the behavior of function by:

$$\text{As } x \rightarrow a^{+/-}, \text{ then } f(x) \rightarrow h$$

The "+" means approaching a from the _____.The "-" means approaching a from the _____.

But don't worry, if I'm approaching _____ or _____ you don't have to mention the direction, I'll find my way!

[Comment](#)
[Like](#)
[See Friendship](#)

Graphing Did you hear that new song, "Teach Me How to Graph"? It's MY JAM! Anyways, thought you might like to know that you can graph $f(x) = \frac{1}{x}$ by using transformations. I've heard someone call this function a _____, but I hate to be mean and call people names.

[Comment](#)
[Like](#)
[See Friendship](#)

Appendix I:
Concept Presentation Artifacts

Group #2 – Hank, Mabel, Scott, Joy PowerPoint Slides

The Triangle Inequality

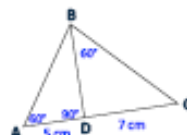
Objectives:

1. SWBAT describe the relationship that exists between the three side lengths in a triangle.
2. SWBAT determine whether it is possible to make a triangle given three side lengths.
3. SWBAT find the maximum and minimum allowable values for the third side of a triangle when given the other two sides.

Agenda:

1. Do Now
2. Pair Activity: The Triangle Inequality
3. Classwork

Do Now



1. True or false:
 - a. BD is an angle bisector of $\angle ABC$.
 - b. BD is a median of AC.
 - c. BD is a perpendicular bisector of AC.
 - d. BD is an altitude of $\triangle ABC$.



2. True or false:
 - a. $NP = OP$.
 - b. $RS > RT$.
 - c. $ND = ST$.
 - d. $OP > ST$.

What do you think?

Can you make a triangle with any three sides? Discuss with the person next to you for 2 minutes:

- Can you make a triangle with sides 1 inch, 1 inch, and 1000 feet?
- Can you make a triangle where all three sides are 1 inch?
- Can you make a triangle with sides 1 inch, 2 inches, and 3 inches?
- When do you think you can make a triangle and when do you think you can't?

Pair Activity: The Triangle Inequality

Each pair should get 5 straws:

Brown stripe = 2 inches
Orange stripe = 4 inches
Black stripe = 7 inches

Purple stripe = 3 inches
Blue stripe = 5 inches

See if you can make a triangle with the following combinations of straws.

Make sure the pointed tips are touching!

1. 2 in., 3 in., and 4 in.
2. 2 in., 4 in., and 7 in.
3. 3 in., 5 in., and 7 in.
4. 2 in., 3 in., and 5 in.
5. 2 in., 5 in., and 7 in.

Do you see a pattern? When can three sides make a triangle?

The Triangle Inequality

Can you make a triangle?

1. 2 in., 3 in., and 4 in.
2. 2 in., 4 in., and 7 in.
3. 2 in., 4 in., and 7 in.
4. 2 in., 5 in., and 7 in.
5. 3 in., 5 in., and 7 in.

In a triangle, the longest side must be LESS THAN (and not equal to) the sum of the other two sides.

The Triangle Inequality

Two sides of a triangle are 6 cm and 4 cm. The third side is a whole number. What is the smallest the third side could be? What is the biggest the third side could be?

What if the third side did not have to be a whole number?

Group #2
PowerPoint slides (continued)

Classwork

Is it possible for a triangle to have the sides with the lengths indicated? Explain.

1. 3, 7, 8
2. 1, 1, 4
3. 9, 6, 2

4. Two sides of a triangle have lengths 10 and 15. The third side can be any number between _____ and _____.

5. A player kicks a soccer ball straight ahead at 40 mph. On the field, there is a 9 mph wind that is blowing at an angle to the kicked ball. The final speed of the ball will be equal to the third side of a triangle where the other two sides are 40 and 9.

The final speed of the ball will be between _____ and _____.

Summary Question

1. Two sides of a triangle have lengths x and y . Find the range of possible values for the length of the third side, and write two sentences explaining your answer.
2. If you know the lengths of three line segments, explain how to figure out if it is possible to form a triangle from those three line segments.

Group #2
Handout

The Triangle Inequality

Is it possible for a triangle to have the sides with the lengths indicated? Explain.

1. 3, 7, 8

2. 1, 1, 4

3. 9, 6, 2

4. Two sides of a triangle have lengths 10 and 13. The third side can be any number between

_____ and _____.

5. A player kicks a soccer ball straight ahead at 40 mph. On the field, there is a 9 mph wind that is blowing at an angle to the kicked ball. The final speed of the ball will be equal to the third side of a triangle where the other two sides are 40 and 9.

The final speed of the ball will be between _____ and _____.

Group #3: Natalie, Rhett, Tom, Fabian
Opening Exercise

Do Now

Write each number as a product of its prime factors.

36

$$2 \times 2 \times 3 \times 3$$
$$(2^2)(3^2)$$

40

$$2 \times 2 \times 2 \times 5$$
$$(2^3) \times 5$$

75

$$5 \times 5 \times 3$$
$$(5^2) \times 3$$

52

$$3 \times 3 \times 3 \times 2$$
$$(3^3) \times 2$$

21

$$7 \times 3$$
$$7 \times 3$$

38

$$19 \times 2$$
$$19 \times 2$$

15

$$5 \times 3$$
$$5 \times 3$$

67

$$67$$
$$67$$

Now simplify each product using exponents.

Group #3
Handouts

Name: _____ Date: _____

M&M mmmmmmmmath

I am offering you 1000 M&M's today that you can eat in my class for the next two weeks. OR I will give you 2 M&M's today, 4 tomorrow, 8 on Wednesday, and continue this doubling pattern every day for the next two weeks.

Option 1	Option 2
1000 M&M's today	2 M&M's today, 4 M&M's tomorrow, 8 M&M's Wednesday, etc. (doubling every day)

Take a couple minutes to decide which option you would choose, and then after you have decided, come up to the board and place a tally in the appropriate column. When you return to your seat explain to your neighbor which option you chose and why.

Group #3
Handouts (continued)

Now, use your calculators to figure out how many M&M's you will receive on each day if you choose option 2 (doubling every day).

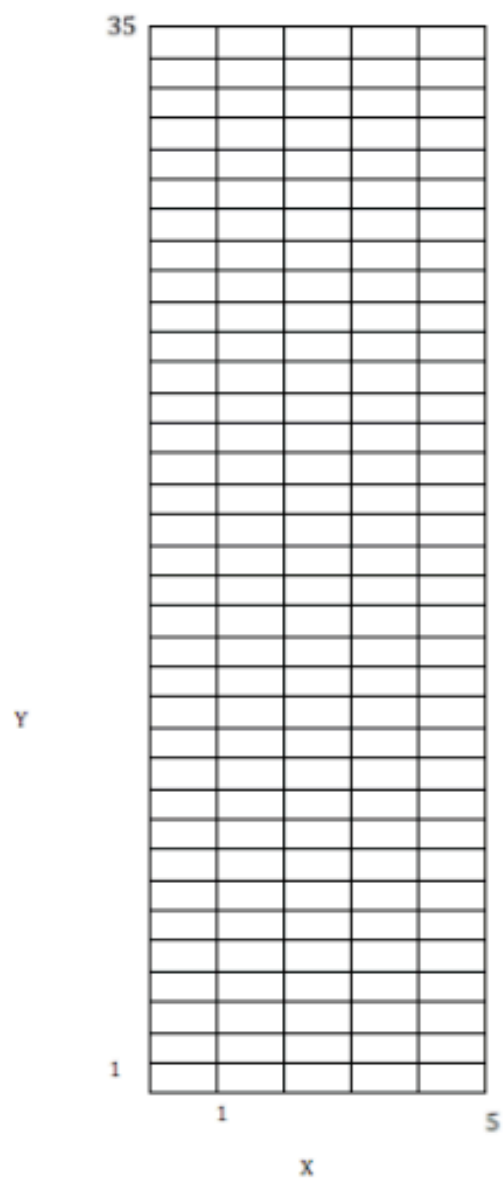
Day #	Number of M&M's received	Write each number as a product of its prime factors	A simplified version of this number
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			

Can you figure out the equation of the relationship between the number of M&M's received and the day?

Number of M&M's received = _____

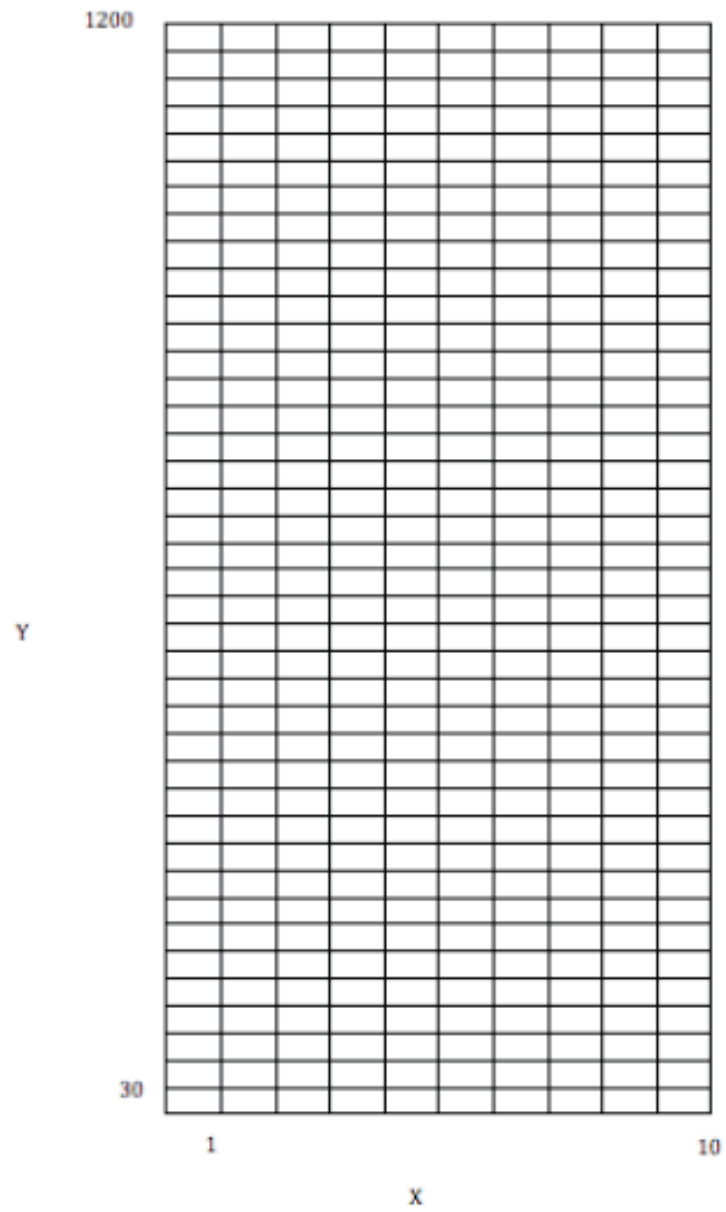
Group #3
Handouts (continued)

Graph your results for days 1-5



Group #3
Handouts (continued)

Graph your results for days 1-10



Group #3
Handouts (continued)

Challenge Problems

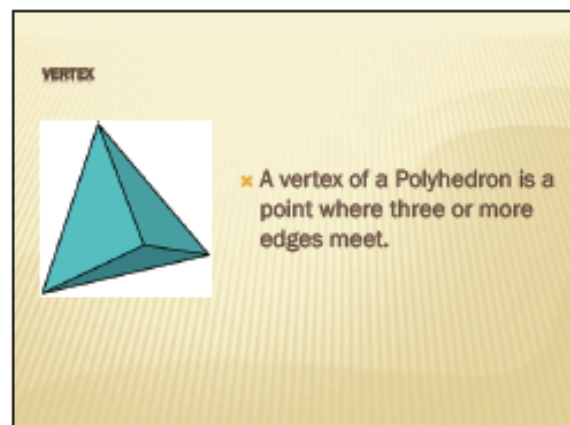
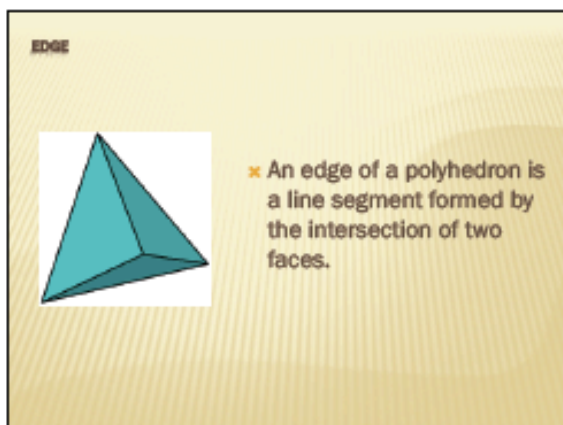
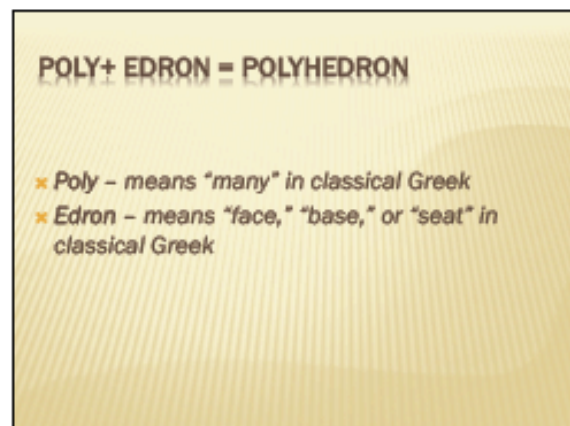
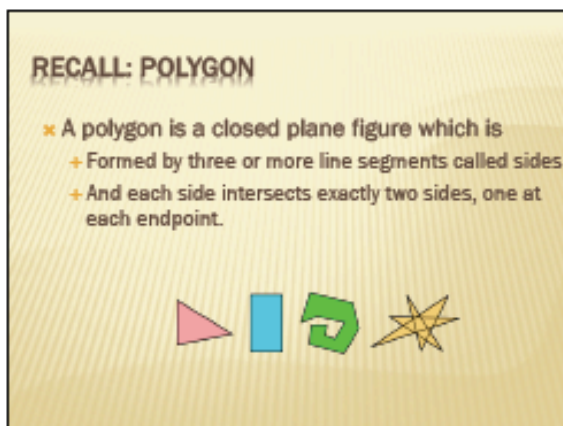
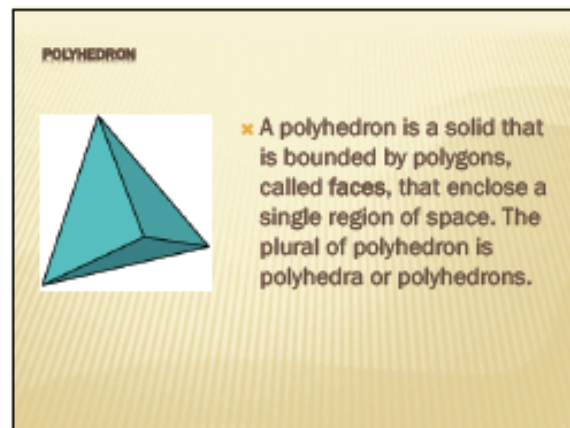
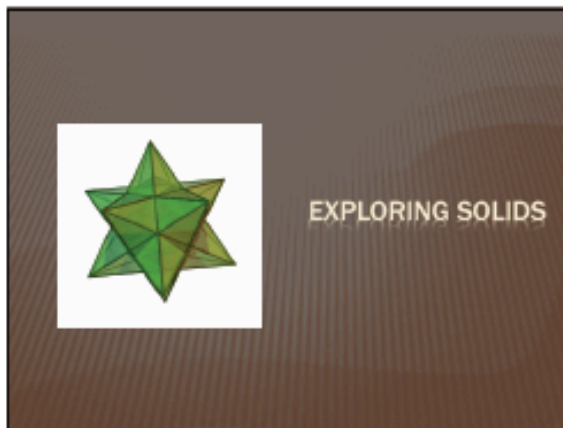
How would the equation be different if I tripled the number of M&M's I gave you each day?

How would the equation be different if I gave you 3 M&M's on the first day, but then doubled the number of M&M's I gave you each day?
3, 6, 12, 24, ...

FOOD FOR THOUGHT



- This type of math is used when calculating the value of bank accounts using compound interest.
- Exponential growth is used in determining population sizes
- Bacteria grow exponentially

Group #4 – Bryce, Kris, unnamed
PowerPoint slides




Group #4
PowerPoint slides (continued)

POLYHEDRA VS. NON-POLYHEDRA

- ✖ Polyhedra
 
- ✖ Not polyhedra
 

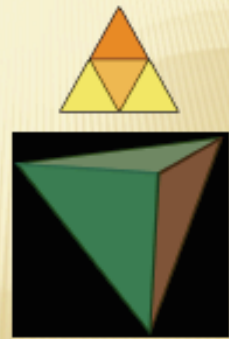
REGULAR POLYHEDRA

- ✖ A regular polyhedron is a polyhedron whose faces are congruent regular polygons
- ✖ Recall:
 - + A regular polygon is a polygon that is both equilateral and equiangular



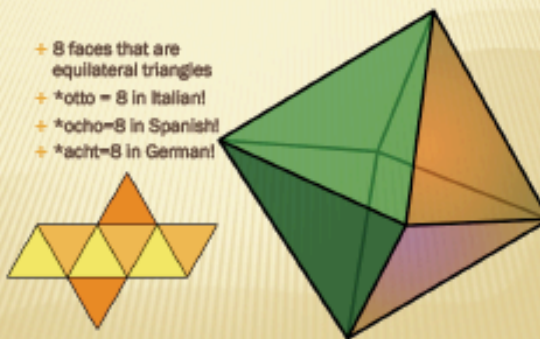
EXAMPLES OF REGULAR POLYHEDRA

- ✖ Tetrahedron
 - + 4 faces that are equilateral triangles




OCTAHEDRON

- + 8 faces that are equilateral triangles
- + *otto = 8 in Italian!
- + *ocho = 8 in Spanish!
- + *acht = 8 in German!




DODECAHEDRON

- 12 Faces which are regular pentagons
- *dodeca = 12 in Greek!
- *dodici = 12 in Italian!



ICOSAHEDRON

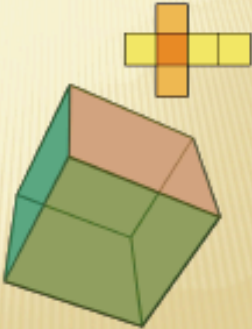
- 20 faces that are equilateral triangles.
- *ekosi = 20 in Greek!



Group #4
PowerPoint slides (continued)

THE CUBE!

- ✧ A cube is a regular polyhedron with square faces



CONSTRUCTING POLYHEDRA

	# Vertices	# Edges	# Faces	Euler's Characteristic
Tetrahedron				
Cube				
Dodecahedron				

Formula:


EULER'S CHARACTERISTIC

$$V - E + F$$

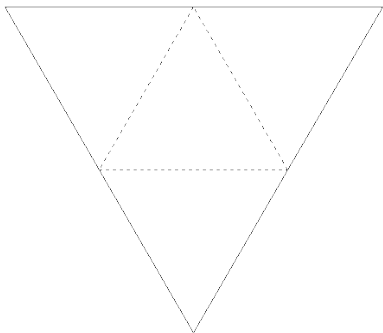
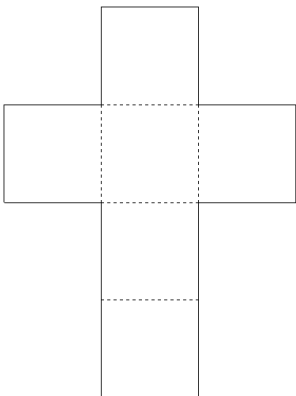
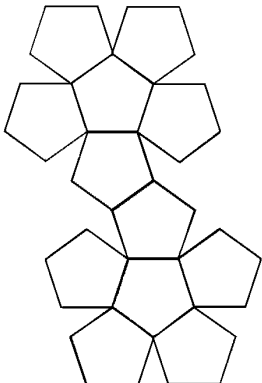
- ✧ Euler's Characteristic is a way for us to tell if a shape is a regular polyhedron, without having to draw or construct the shape.
- ✧ If the Euler Characteristic for your shape is 2, then you have a regular polyhedron.

TRY THIS OUT!

- ✧ Construct a *pyramid* using the picture to the right.
 - ✧ Is this a regular polyhedron?
- ✧ Calculate Euler's Characteristic.
- ✧ What do you notice if your shape is NOT a regular polyhedron?



Group #4
Handout

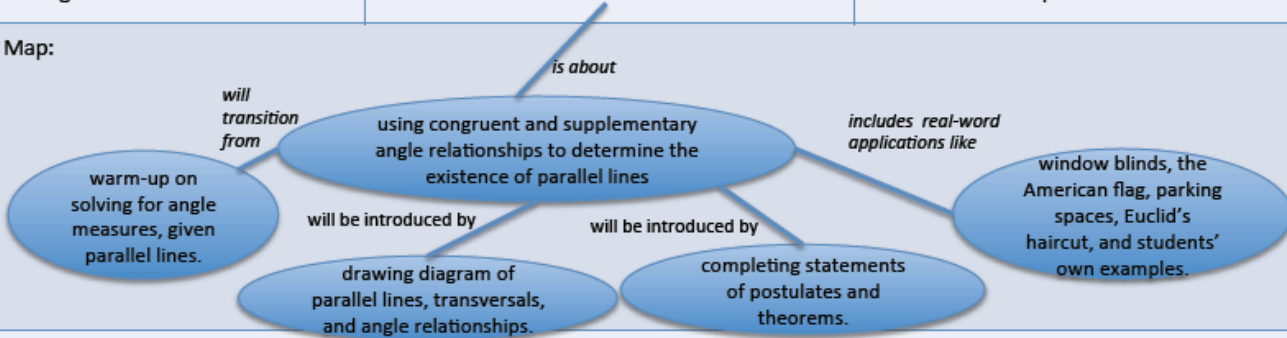
Shape	# Vertices	# Edges	# Faces	Euler's Characteristic
<p>Tetrahedron</p> 				
<p>Cube</p> 				
<p>Dodecahedron</p> 				

Formula for Euler's
Characteristic:

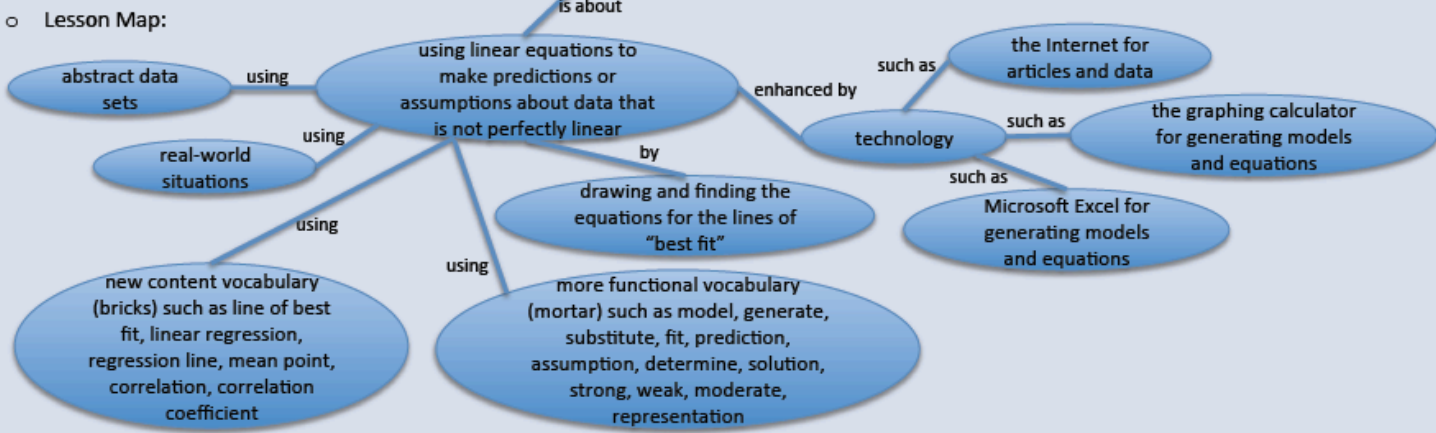
Pyramid				
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Appendix J:
Lesson Organizer Samples

Sample Lesson Organizer #1

Unit: Parallel Lines		
<ul style="list-style-type: none">Lesson objectives:<ul style="list-style-type: none">Content: SWBAT relate what they learned about converse statements to theorems and postulates about proving lines are parallel and use angle relationships to solve algebraic problems and prove that lines are parallel.Language: With a partner, SWBAT create and display a hierarchical graphic organizer to differentiate the angle pairs that are congruent from those that are supplementary. Students will discuss the posted organizers as class and peer-correct them.		
<ul style="list-style-type: none">Relationships: Parallel lines, transversals, angle relationships such as vertical, supplementary, alternate interior, and corresponding	<ul style="list-style-type: none">Lesson Topic: Proving parallel lines, given angle measures	<ul style="list-style-type: none">Task-related Strategies<ul style="list-style-type: none">Diagramming and highlightingCreating hierarchical graphic organizersClass presentations
<ul style="list-style-type: none">Lesson Map:<div></div>		
<ul style="list-style-type: none">Challenge Question(s): Given a diagram and the fact that $r \parallel s$ and angle 1 = angle 3, prove that $p \parallel q$.		
<ul style="list-style-type: none">Self-test Question(s):<ol style="list-style-type: none">What are the converse statements of the postulate and theorems you learned last class?How can you determine which lines are parallel, given diagrams and angle relationships?How do you find the value of x that makes specific lines parallel?	<ul style="list-style-type: none">Task(s):<ol style="list-style-type: none">Complete worksheet: determine which lines are parallel, solve algebraic equations to find angle measures that make lines parallel, and justify all answers with a postulate or theorem.Complete a proof showing that lines are given information about their angles.	
<ul style="list-style-type: none">Assessment task(s): (Observation)<ol style="list-style-type: none">Are students able to determine which lines are parallel?Can they solve algebraic equations to find angle measures?Are students effectively using postulates or theorems to justify their answers?Can students complete their note pages, using drawings of diagrams and appropriate statements of postulates and theorems?		

Sample Lesson Organizer #2

Unit: Displays of data		
<ul style="list-style-type: none">Lesson objectives:<ul style="list-style-type: none">Content: SWBAT generate lines of best fit for real-world data, determine the strength of the line's representation of the data, and use the line to make predictions or assumptions use the line of best fit.Language: SWBAT create a sequential graphic organizer in small groups, outlining the general process of linear regression, then write 1-2 paragraphs describing the process independently.		
<ul style="list-style-type: none">Relationships: Coordinate planes, scatter plots, linear functions, variables	<ul style="list-style-type: none">Lesson Topic: Linear Regression	<ul style="list-style-type: none">Task-related Strategies<ul style="list-style-type: none">Creating surveysCreating tables and other displays of data
<ul style="list-style-type: none">Lesson Map:		
<ul style="list-style-type: none">Challenge Question(s):<ol style="list-style-type: none">Can the process of regression be used for other kinds of mathematical relationships (i.e. exponential, quadratic, cubic, etc.)?What is the difference between linear regression and linear extrapolation?		
<ul style="list-style-type: none">Self-test Question(s):<ol style="list-style-type: none">What is linear regression?Given a set of data, what are the criteria for performing linear regression?How can I prove that my regression line is a good representation of my data?	<ul style="list-style-type: none">Task(s): Text assignment(s): problems generating lines of best fit for various data sets, testing their strength, interpreting the results, and making calculation-based claims	
<ul style="list-style-type: none">Assessment Task(s): (summative) Group survey project & presentation: Students will work in groups to choose a topic, survey at least twenty people, and generate displays to interpret and present their findings.		